Math 511

~~Q's~~ $B_1 = [x, 1]$ $B_2 = [2x-1, 2x+1]$ in P_2

$B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $B_2 = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$ ← x's ← const.

a) from B_2 to B_1 $[c]_{B_1} = B_1^{-1} B_2 [c]_{B_2}$

transition matrix is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$

b) from B_1 to B_2

transition matrix is $\begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}^{-1} = \boxed{?}$

3.6 #6 $\boxed{A\mathbb{x} = \mathbb{b}}$ if $\mathbb{b}$ is in col. space of A...

→ we know we have a sol'n (1 or ∞?)

... col. vectors of A are $\boxed{\text{linearly dep.}}$

$\boxed{\text{ex}}$ $A \xrightarrow[\text{ops}]{\text{row}} U = \begin{bmatrix} \boxed{1} & 2 & 0 & 3 & 0 \\ 0 & 0 & \boxed{1} & 1 & 0 \\ 0 & 0 & 0 & 0 & \boxed{1} \end{bmatrix}$

$u_{12} = 2u_{11}$
$u_{14} = 3u_{11} + u_{13}$ → $\boxed{\begin{array}{l} a_{12} = 2a_{11} \\ a_{14} = 3a_{11} + a_{13} \end{array}}$

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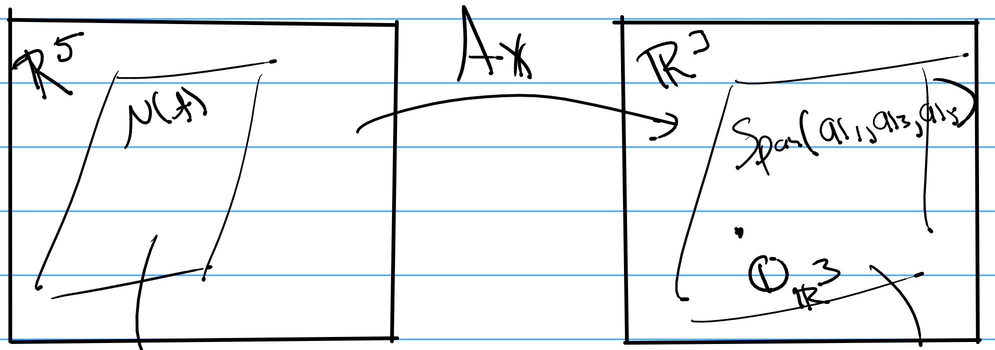
linear dep. eqn's means you have free vars.

→ $\boxed{\infty \text{ Sol'n's}}$

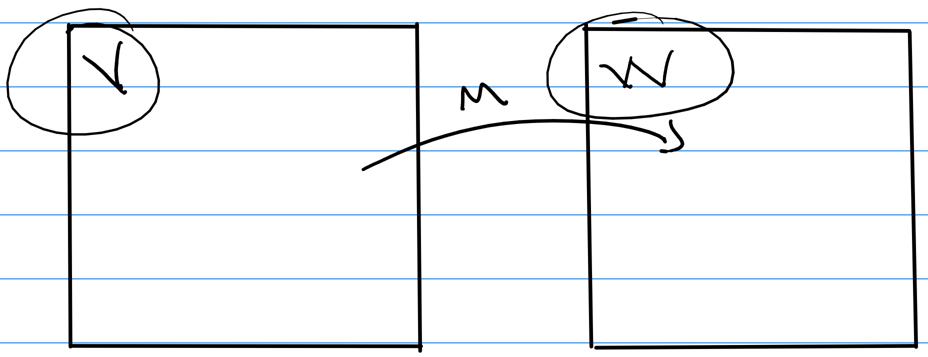
Note: A is 3×5 $\xrightarrow[\text{ops}]{\text{row}}$ $U = \begin{bmatrix} 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$



$\dim(N(A)) = 2$ $\dim(\text{Col Space}) = 3$

$\underset{3\times 5}{A}\ \underset{5\times 1}{\mathbf{x}} = \mathbf{y}$

Ch 4



M is a mapping from V to W

$M(\mathbf{v}) = \mathbf{w}$

$\mathbf{v} \in V$, $\mathbf{w} \in W$

ex of mappings

$x^2 + 2x \xrightarrow{M = \text{deriv}} 2x + 2$

$2x + 2 \xrightarrow{\text{anti-deriv}} x^2 + 2x + C$

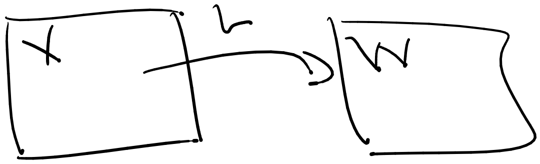
$2x+2$ over $[0,1]$ $\xrightarrow{}$ def. int. 3 b/c $\int_0^1 2x+2\,dx = x^2+2x\Big|_0^1 = 3$

Idea came from $\mathbf{y} = A\mathbf{x}$ type of mappings.

→ Consider a specific type of mapping..

is Linear Transformation / Linear Operator on V

Def



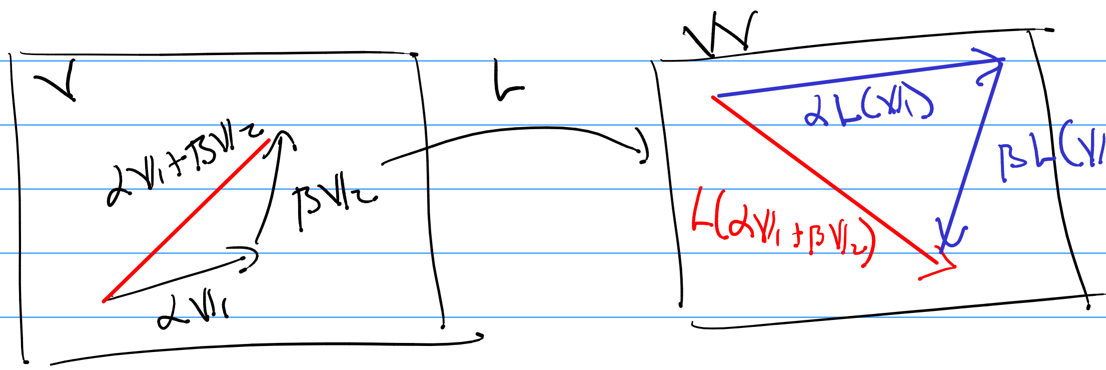
Notation: $L: V \to W$

$L(v) = w$

L is a Linear Transformation
(Linear operator on V)

if given $v_1, v_2 \in V$, $\alpha, \beta \in \mathbb{R}$

$$L(\alpha v_1 + \beta v_2) = \alpha L(v_1) + \beta L(v_2)$$



Probs §4.1 given a mapping is it a Linear Operator?

① show $L(\alpha v_1 + \beta v_2) = \alpha L(v_1) + \beta L(v_2)$

or

② show i) $L(v_1 + v_2) = L(v_1) + L(v_2)$

and

ii) $L(\alpha v_1) = \alpha L(v_1)$

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ex $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 3 \end{bmatrix}$ $A: \mathbb{R}^3 \to \mathbb{R}^2$

by $y = Ax$ $x \in \mathbb{R}^3$

$y \in \mathbb{R}^2$

is $y = Ax$ a linear operator?

Show: ① $A(x_1 + x_2) = Ax_1 + Ax_2$ true b/c of distrib. of Matrix mult.

② $A(\alpha x_1) = \alpha A x_1$ true b/c assoc. prop. of scalar matrix mult.

these are always true for all A. So any $y = Ax$ is a linear operator.

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ex $D : C[a,b] \to C[a,b]$

D is the 1^{st} Derivative. is D a linear operator?

Show: ① $D(f_1 + f_2) \stackrel{?}{=} D(f_1) + D(f_2)$

② $D(\alpha f_1) \stackrel{?}{=} \alpha D(f_1)$

So ① $\frac{d}{dx}[f_1(x) + f_2(x)] = f_1'(x) + f_2'(x)$

true

b/ sum rule of deriv.

② $\frac{d}{dx}[\alpha f_1(x)] = \alpha \frac{d}{dx}[f_1(x)]$ ~~$\neq 0 \cdot f_1(x)$~~

true by scalar times function rule or prod. rule

∴ Deriv. is a Linear operator on $C[a,b] \to C[a,b]$

Properties of $L : V \to W$

① $L(0_v) = 0_w$

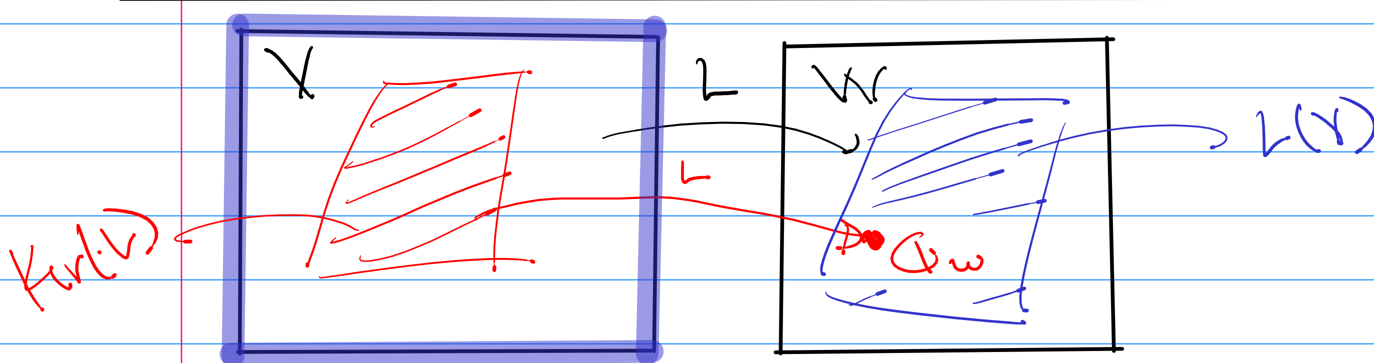
② $\alpha_i \in \mathbb{R}$ $\quad V_i \in V$

$$L(\alpha_1 V_1 + \alpha_2 V_2 + \dots + \alpha_n V_n) = \alpha_1 L(V_1) + \alpha_2 L(V_2) + \dots + \alpha_n L(V_n)$$

③ $L(-V) = -L(V)$

④ <u>Def:</u> Identity Linear Operator

$$I(V) = V \quad \underline{\text{with}} \quad I : V \to V$$



① Kernel of L, $\mathrm{Ker}(L) = \{ V \in V \mid L(V) = 0_w \}$

② Range of L, $L(V) = \{ W \in W \mid$ there is a V $L(V) = W \}$

③ S is a subspace of V

$$L(S) = \{ W \in W \mid V \in S, \; L(V) = W \}$$