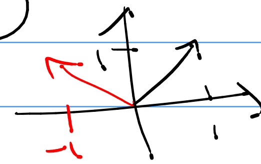
Math 511

Q's 4.3 (2) $U = \begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}$ $V = \begin{bmatrix}2 & 1\\ 1 & 0\end{bmatrix}$ given

$L: \mathbb{R}^2 \to \mathbb{R}^2$ $L\begin{pmatrix}x_1\\ x_2\end{pmatrix} = \begin{bmatrix}-x_1\\ x_2\end{bmatrix}$



i) is L a linear transform on $\mathbb{R}^2 \to \mathbb{R}^2$?

show $L(\alpha_1 v_1 + \alpha_2 v_2) = \alpha_1 L(v_1) + \alpha_2 L(v_2)$

or

show (1) $L(v_1 + v_2) = L(v_1) + L(v_2)$

(2) $L(\alpha v_1) = \alpha L(v_1)$

$$L(\alpha_1 \underset{\begin{bmatrix}a\\ b\end{bmatrix}}{\underset{\shortparallel}{v_1}} + \alpha_2 \underset{\begin{bmatrix}c\\ d\end{bmatrix}}{\underset{\shortparallel}{v_2}}) = L\left(\begin{bmatrix}\alpha_1 a + \alpha_2 c\\ \alpha_1 b + \alpha_2 d\end{bmatrix}\right) = \begin{bmatrix}-\alpha_1 a - \alpha_2 c\\ \alpha_1 b + \alpha_2 d\end{bmatrix}$$

$$= \alpha_1 \begin{bmatrix}-a\\ b\end{bmatrix} + \alpha_2 \begin{bmatrix}-c\\ d\end{bmatrix}$$

$$= \alpha_1 L(v_1) + \alpha_2 L(v_2) \quad \checkmark$$

ok!

ii) Matrix rep. of $\boxed{\text{linear Transform}}$

a) Standard $A_E = \left[L\begin{pmatrix}1\\ 0\end{pmatrix} \; L\begin{pmatrix}0\\ 1\end{pmatrix}\right] = \begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}$

b) basis $U = \begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}$

$$Y_u = (U^{-1} A_E U) X_u$$

$$A_u = \begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}^{-1} \begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix} \begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}$$

c) basis $V = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$

$$A_V = V^{-1} A_E V = \underline{\underline{\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}}}$$

iii) coord in U into coord in V ?

$$X_V = V^{-1} U X_u$$

$$S = V^{-1} U = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = ?$$

transition from U coord to V coord matrix

iv) V coord to U coord

$$S^{-1} = U^{-1} V$$

#2 c ① $L(V_1) = a_{11} V_1 + a_{21} V_2$ $\qquad L\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -x_1 \\ x_2 \end{bmatrix}$

② $L(V_2) = a_{12} V_1 + a_{22} V_2$

$$A = A_V = \begin{bmatrix} 1 & 0 \\ -4 & -1 \end{bmatrix} \qquad V_1 = \begin{matrix} 2 \\ 1 \end{matrix}$$

$A_E X_E = L(X)$ standard

$A_V X_V = L(V)$ V basis

$$L(V_1) = L\left(\begin{pmatrix} 2 \\ 1 \end{pmatrix}_E\right) = \begin{pmatrix} -2 \\ 1 \end{pmatrix}_E$$

$$A_E (V_1) = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}_E$$

$v_1 = 1 \cdot v_1 + 0 v_2 = 2e_1 + 1e_2$

$A_V x_V =$

$v_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}_E = \begin{bmatrix} 1 \\ 0 \end{bmatrix}_V$

$v_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}_E = \begin{bmatrix} 0 \\ 1 \end{bmatrix}_V$

Exam 2 — 11 probs @ 10 pts each
100 pts = 100%

3.1 Vector Spaces (1 prob)

① Is it a vector space? (I will give you the axiom sheet)

objects, $v_1 + v_2$, αv_1 ← defined

$\mathbb{R}^n$, $\mathbb{R}^{n \times n}$, P_n, $C[a,b]$, $C^{(n)}[a,b]$

3.2 Subspaces (1 prob)

① Is it (are they) subspaces?

3.3 Linear Independence (2 prob)

① are the vectors from $\mathbb{R}^n$ linearly ind?
② are the polynomials from P_n linearly ind?

3.4 Basis / Dimensions (1 prob)

① given vectors of $\mathbb{R}^3$

a) dim (span(given vectors))
b) pare down, extend to create a basis for $\mathbb{R}^3$

3.5 change of Basis (1 prob)

① given bases B_1, B_2

a) - find matrix to change from coord of B_1 to B_2

b) - find matrix to change from coord of B_2 to B_1

ex a) $\boxed{B_2^{-1} B_1} X_{B_1}$

3.6 Row / Col space (1 prob)

① given $A \longrightarrow$ find U (reduced row ech.)

- rank?
- nullity?
- dep. eqn's for U? for A?
- col space = span(?)
- row space = span(?)
- $N(A)$ = span(?)

4.1 Linear Operators (2 prob)

① $L: \mathbb{R}^n \to \mathbb{R}^n$ is it a linear operator?

② given a linear operator on P_n or subspace of $C[a,b]$

$\ker(L)$?

$L(V)$?

4.2 Matrix Rep. of Linear Operators (1 prob)

1. $\mathbb{R}^n$ find A_E for $L: \mathbb{R}^n \to \mathbb{R}^n$

4.3 <u>Similarity</u> (1 prob)

1. P_n given
   1. standard basis
   2. basis B_1

find A_E for L
find A_{B_1} for L