Math 511

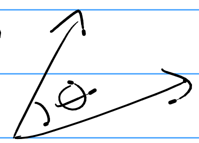
Q's/ $\mathbb{R}^2, \mathbb{R}^3$ $\boxed{X^T Y}$ is used to find length's and angle type structure

$\quad\quad = \cos\theta$.

length ① $\|X\|^2 = X^T X$

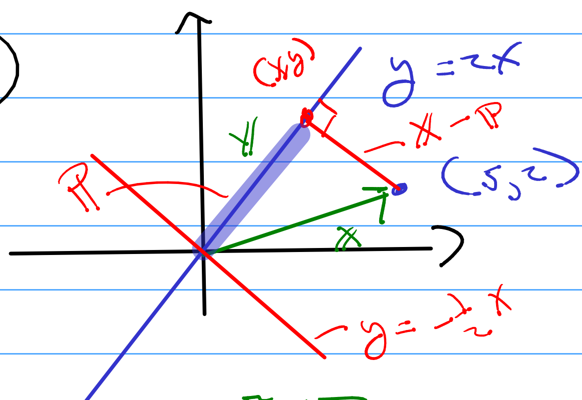
or

$\|X\| = (X^T X)^{1/2}$

② 

$$\cos\theta = \frac{X^T Y}{\|X\|\,\|Y\|}$$

5.1 #5



try:

$V = \begin{bmatrix} x-5 \\ y-2 \end{bmatrix}$

$u = \begin{bmatrix} x \\ y \end{bmatrix}$

find x, y so

$V^T u = 0$?

so $P = \begin{bmatrix} x \\ y \end{bmatrix}$ so that

(x,y) is closest to (5,2)

where P is proj. of $\underbrace{\begin{bmatrix} 5 \\ 2 \end{bmatrix}}_{X}$ onto [any vector on y=2x]

ex $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ Y

$$P = \frac{\begin{bmatrix} 5 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}}{\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}} \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \frac{X^T Y}{Y^T Y}\, Y$$

Exam 2

$\alpha \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ \alpha x_2 \end{bmatrix} \in \mathbb{R}^2$

C1/C2 are OK

A5/A8 A6 $(\alpha+\beta)\mathbb{X} = \alpha\mathbb{X} + \beta\mathbb{X}$

$\begin{matrix} x_1 \\ (\alpha+\beta)x_2 \end{matrix} \overset{?}{\neq} \begin{matrix} x_1 \\ \alpha x_2 \end{matrix} + \begin{matrix} x_1 \\ \beta x_2 \end{matrix}$

$-\mathbb{X} = +(-1)\mathbb{X}$

$\mathbb{X} + (-1)\mathbb{X} = \begin{matrix} x_1 \\ x_2 \end{matrix} + \begin{matrix} x_1 \\ -x_2 \end{matrix} = \begin{matrix} 2x_1 \\ 0 \end{matrix} \neq \begin{matrix} 0 \\ 0 \end{matrix}$

2a) P_4 any $p = a + bx + cx^2 + dx^3$

Subset all p so that $\boxed{p(0) = 0}$

$p = bx + cx^2 + dx^3$

$\boxed{\text{Subspace:}}$ $\boxed{\text{C1, C2, and A3}}$ Zero is in subset.

$z(x) = 0 + 0x + 0x^2 + 0x^3$

assume $p(0) = 0 \rightarrow$ show $(\alpha p)(0) = 0$

$\begin{matrix} p(0) = 0 \\ q(0) = 0 \end{matrix} \rightarrow$ show $(p+q)(0) = 0$

2b subset $A = \begin{bmatrix} a & b \\ c & 1 \end{bmatrix}$

zero $= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \notin$ Subset

(3) $X = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 4 & 2 \\ -1 & 0 & 1 \end{bmatrix}$ use $\det(X) = 0$

$\rightarrow$ dep

(4) $\{(1), (1+x), (x+x^2), (x^2+1)\}$ P_3

$\underline{A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}}$ $\rightarrow$ show free var and a soln.

$c_1 1 + c_2(1+x) + c_3(x+x^2) + c_4(x^2+1) = 0 + 0x + 0x^2$

(5) $X = \begin{bmatrix} 1 & 2 & 1 & 3 \\ 2 & 5 & 3 & 7 \\ 1 & 0 & -1 & 1 \end{bmatrix} \rightarrow U = \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\dim\left(\mathrm{span}\left(\begin{smallmatrix}1\\2\\1\end{smallmatrix}, \begin{smallmatrix}2\\5\\0\end{smallmatrix}\right)\right) = 2$

basis $\mathbb{R}^3$ $\rightarrow$ find one vector more

(6) $B_1 = \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}$ $B_2 = \begin{bmatrix} 1 & 2 \\ -3 & 7 \end{bmatrix}$ $\leftarrow$ result $\leftarrow$ x's

$S = B_1^{-1} B_2 = \boxed{\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 2 \\ -3 & 7 \end{bmatrix}}$

e.R

(7) show linear operator:

(1) $L(\alpha V) = \alpha L(X)$

(2) $L(V_1 + V_2) = L(V_1) + L(V_2)$ NO

$L\begin{pmatrix} \alpha x_1 \\ \alpha x_2 \end{pmatrix} = \begin{matrix} \alpha x_1 \\ \alpha x_1 \\ \alpha x_1 + \alpha^2 x^2 \end{matrix} \neq \begin{matrix} \alpha x_1 \\ \alpha x_1 \\ \alpha(x_1 + x_2^2) \end{matrix}$

8 $\begin{array}{cccc} \begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ a_1 \quad\quad a_3 \end{array}$ $a_4 = a_1 + 2a_3$

$a_2 = -2a_1$

dep. write a's

9 P_3 $\quad p = \boxed{a} + bx + cx^2$

$L(p) = xp' + p'' = x(b + 2cx) + (2c)$

$L(p) = \underline{2c} + \underline{b} \cdot x + \underline{2c} \cdot x^2$

$L(V) = P_3$

$Ker(L) \to$ when is $2c + bx + 2cx^2 = 0 + 0x + 0x^2$

$c = 0,\ b = 0,\ \underline{a = \text{anything}}$

$= P_1$

$\mathbb{R}^2 \to \mathbb{R}^3$

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$A_E = [L\begin{pmatrix}1\\0\end{pmatrix}\ L\begin{pmatrix}0\\1\end{pmatrix}] = ?$

11 $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$A_E = [L\begin{pmatrix}1\\0\\0\end{pmatrix}\ L\begin{pmatrix}0\\1\\0\end{pmatrix}\ L\begin{pmatrix}0\\0\\1\end{pmatrix}] = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$xp' + p''$

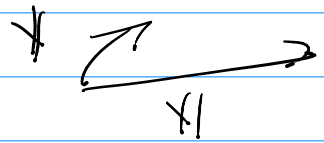
$A_B = B^{-1} A_E B = ?$

5.1 $\mathbb{R}^2, \mathbb{R}^3$

$x^T y$ Scalar product can be use to find ...

① length $\|x\| = (x^T x)^{1/2}$

② angle compute $\cos\theta = \dfrac{x^T y}{\|x\| \, \|y\|}$



$\mathbb{R}^n$ $x^T y = x_1 y_1 + x_2 y_2 + x_3 y_3 + \dots + x_n y_n$

① $\|x\|^2 = x^T x$

② $|x^T y| \le \|x\| \, \|y\|$

$$-1 \le \frac{x^T y}{\|x\| \, \|y\|} \le 1$$

③ let $\cos\theta = \dfrac{x^T y}{\|x\| \, \|y\|}$

④ call $x \perp y$ (orthogonal)

if $x^T y = 0$

V is a vector space $(\mathbb{R}^n)$

and $\mathbb{x}^T \mathbb{y}$ allows us to study magnitude and directions within $\mathbb{R}^n$

→ Subspaces of V

Subset of V that has ...

1. $0 \in S$
2. $v \in S \Rightarrow \alpha v \in S$
3. $v_1, v_2 \in S \Rightarrow v_1 + v_2 \in S$

ex $\mathbb{R}^4$

$$S = \text{Span}\left(\begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\\0\end{bmatrix}\right) = \text{Span}(e_1, e_3)$$

$$\dim(S) = 2$$

Compare to

$$T = \text{Span}\left(\begin{bmatrix}0\\1\\0\\0\end{bmatrix}\right) = \text{Span}(e_2)$$

$$\dim(T) = 1$$

any $s \in S$ $\quad s = a \cdot e_1 + b \cdot e_3 = \begin{bmatrix}a\\0\\b\\0\end{bmatrix}$

$t \in T$ $\quad t = c \cdot e_2 = \begin{bmatrix}0\\c\\0\\0\end{bmatrix}$

Notice: $s^T t = [a\ 0\ b\ 0]\begin{bmatrix}0\\c\\0\\0\end{bmatrix} = 0 \rightarrow s \perp t$

So every vector in S is $\perp$ to every vector in T.

Def if every vector of S is $\perp$ to every vector of T

$\to$ call S, T to be <u>orthogonal</u> subspaces

$$x \perp y \;\Rightarrow\; x^T y = 0$$

$X \perp Y$, means for all $x \in X$ and all $y \in Y$
$$x^T y = 0$$

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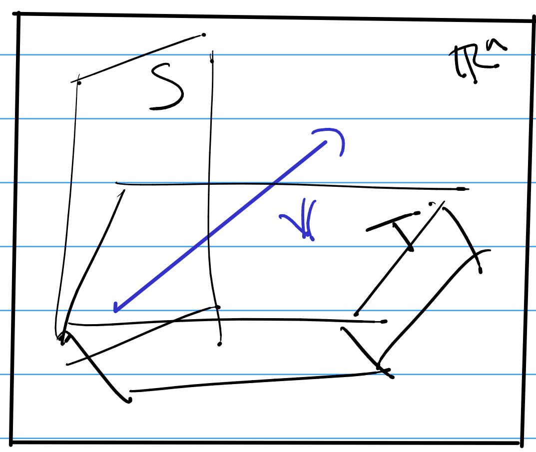
goal



① S is a subspace of $\mathbb{R}^n$

② T is a subspace of $\mathbb{R}^n$

③ $S \perp T$

④ $v \in \mathbb{R}^n$

$$as + bt = v$$