Math 511

~~Q's~~ Sol #16 $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ 2x2

$\det(A - \lambda I) = 0$ show $\operatorname{tr}(A) = a_{11} + a_{22} = \lambda_1 + \lambda_2$

$\boxed{\lambda^2 + b\lambda + c} = 0$ $\det(A) = \lambda_1 \lambda_2$

$\boxed{\operatorname{tr}(A) = -b}$, $\boxed{\det(A) = c}$

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4c}}{2}$$

$$\lambda_1 = \frac{-b}{2} + \frac{\sqrt{b^2-4c}}{2} \qquad \lambda_2 = \frac{-b}{2} - \frac{\sqrt{b^2-4c}}{2}$$

show $\lambda_1 + \lambda_2 = -b$

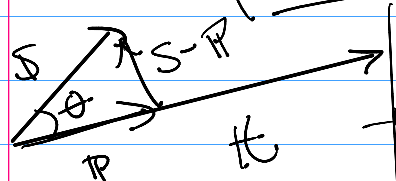
$\lambda_1 \lambda_2 = c$

Exam 3 11 probs @ 10 pts each → 100 pts = 100%

5.1 — 1 problem: Projection type problem ($\mathbb{R}^n$) $\langle v_1, v_2 \rangle = v_1^T v_2$

5.4 — 2 problems: Projection type problems ($C[a,b]$, P_n, $\mathbb{R}^{m \times n}$)
inner product spaces
$\langle v_1, v_2 \rangle =$

s onto t



(1a) $\cos\theta = \dfrac{\langle s, t \rangle}{\|s\| \, \|t\|}$ (1b) $\|v\| = \langle v, v \rangle^{1/2}$

(2) $\|p\| = \|s\| \cos\theta = \dfrac{\langle s, t \rangle}{\|t\|}$

(3) $p = \dfrac{\langle s, t \rangle}{\langle t, t \rangle} t$

(ex) find θ as an arccos for P_3 with

$\langle p, q \rangle$ evaluated at $x=-1$ $x=0$ $x=1$

__between:__ $f_1(x) = 3 + 2x - x^2$ $\quad$ f_1 onto f_2

$f_2(x) = -2 + \pi x + 3x^2$ $\quad$ projection

$$\cos\theta = \frac{\langle f_1, f_2 \rangle}{\|f_1\| \|f_2\|} \to \theta = \arccos\left(\quad\right)$$

$$\langle f_1, f_2 \rangle = f_1(-1) f_2(-1) + f_1(0) f_2(0) + f_1(1) f_2(1)$$

$$= 0 \cdot (1-\pi) + 3 \cdot (-2) + 4 \cdot (1+\pi)$$

$$= 4\pi - 2$$

$$\|f_1\| = 5$$

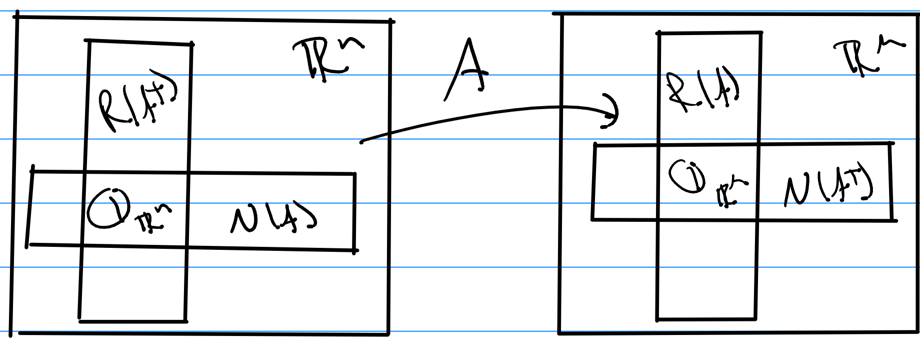
$$\|f_2\| = \left((1-\pi)^2 + 4 + (1+\pi)^2\right)^{1/2}$$

So $$\theta = \arccos\left(\frac{4\pi - 2}{5\sqrt{2\pi^2 + 6}}\right)$$

P = proj. of f_2 onto f_1 ?

$\boxed{5.2}$ Understand and ask questions about, $A_{m \times n}$

$A \to U$



$N(A) = R(A^T)^\perp$ $\qquad$ $N(A^T) = R(A)^\perp$

$A_{5\times4}$ if $\begin{bmatrix}1\\-1\\0\\0\end{bmatrix} \in N(A)$ is $[1\ 1\ 1\ 0]$ in the row space of A?

$\begin{bmatrix}1\\1\\1\\0\end{bmatrix} \in R(A^T)$?

$\boxed{5.3}$ data → least squares fit.

ex

x	y
-2	0
-1	3
0	1
1	2
2	-1

fit with a 'quadratic' $y = ax^2 + bx + c$

$$a(-2)^2 + b(-2) + c = 0$$
$$a(-1)^2 + b(-1) + c = 3$$
$$a(0)^2 + b(0) + c = 1$$
$$a(1)^2 + b(1) + c = 2$$
$$a(2)^2 + b(2) + c = -1$$

$$\begin{bmatrix}4 & -2 & 1\\ 1 & -1 & 1\\ 0 & 0 & 1\\ 1 & 1 & 1\\ 4 & 2 & 1\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix} = \begin{bmatrix}0\\3\\1\\2\\-1\end{bmatrix}$$

least squares fit

$$\begin{bmatrix}a\\b\\c\end{bmatrix} = \left(\begin{bmatrix}4 & 1 & 0 & 1 & 4\\ -2 & -1 & 0 & 1 & 2\\ 1 & 1 & 1 & 1 & 1\end{bmatrix}\begin{bmatrix}4 & -2 & 1\\ 1 & -1 & 1\\ 0 & 0 & 1\\ 1 & 1 & 1\\ 4 & 2 & 1\end{bmatrix}\right)^{-1}\left(\begin{bmatrix}4 & 1 & 0 & 1 & 4\\ -2 & -1 & 0 & 1 & 2\\ 1 & 1 & 1 & 1 & 1\end{bmatrix}\begin{bmatrix}0\\3\\1\\2\\-1\end{bmatrix}\right)$$

stop

5.5   2 probs   given $u_1, u_2, \ldots, u_k$ orthonormal basis

→ show orthonormal

→ use coord. of orthonormal basis

ex given u_1, u_2, u_3 orthonormal

$u_i \in C[-1,1]$

$\langle f,g\rangle = \int_{-1}^{1} fg\,dx$

$$x = 3u_1 + 2u_2 - u_3$$

$$y = -u_1 - u_2 + 2u_3$$

$$\langle x, y\rangle = (3)(-1) + (2)(-1) + (-1)(2)$$

$$\|x\| = \left(3^2 + 2^2 + (-1)^2\right)^{1/2}$$

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gram schmidt.

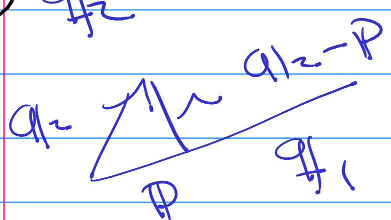
given $a_1, a_2, a_3 \to$ QR factorize

q_1, q_2, q_3

$$\begin{array}{cc} Q & R \\ \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix} & \begin{bmatrix} \|a_1\| & (+) & (1) \\ 0 & (*) & (2) \\ 0 & 0 & (3) \end{bmatrix} \end{array}$$

(1) $q_1 = \dfrac{1}{\|a_1\|} a_1$

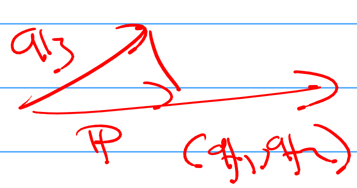
(2) q_2



$$p = \langle a_2, q_1\rangle q_1$$

$$q_2 = \frac{1}{\|a_2 - p\|}(a_2 - p)$$

(3) q_3



$$p = \langle a_3, q_1\rangle q_1 + \langle a_3, q_2\rangle q_2$$

$$q_3 = \frac{1}{\|a_3 - p\|}(a_3 - p)$$

Ch 6 — 6.1 / 6.3 (3 prbs)

① ② given A find λ_i, x_i

③ $\boxed{AX = XD}$

$A = \boxed{XDX^{-1}}$