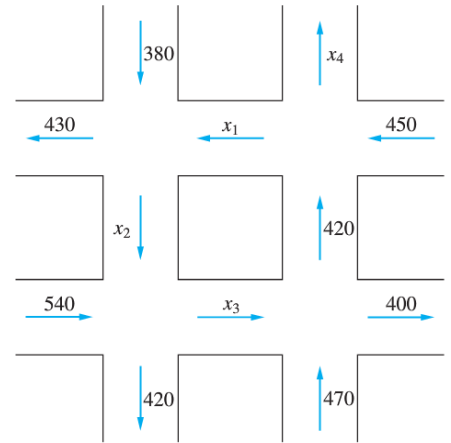MATH 511 - EXAM 1

1) Solve the system of equations. DO NOT use matrices.

$$
\begin{aligned}
2y + 2u &= 8 \\
x - 2y + z + u &= 0 \\
2x + y + z - u &= 3 \\
x + y + u &= 5
\end{aligned}
$$

2) Solve the system of equations. Use Gaussian Elimination on an augmented matrix.

$$\begin{aligned} 2y + 2u &= 8 \\ x - 2y + z + u &= 0 \\ 2x + y + z - u &= 3 \\ x + y + u &= 5 \end{aligned}$$

3) Determine the values of x_i for the traffic flow diagram by using Gauss-Jordan elimination on an augmented matrix.

4) Perform the indicated operations.

a) $\begin{pmatrix} -1 & 1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix}^T - 2 \begin{pmatrix} x & -y \\ y & 2x \end{pmatrix}$

b) $\begin{pmatrix} a & b & c \end{pmatrix}^T \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$

5) Calculate $I + A + A^2$ for the matrix A ...

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

6) Find the LU factorization for the the given matrix.

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 4 & 3 & 1 \\ 2 & 4 & 4 \end{pmatrix}$$

7) Find A^{-1} for the the given matrix.

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 4 & 3 & 1 \\ 2 & 4 & 4 \end{pmatrix}$$

8a) State Theorem 1.3.1

b) State Theorem 1.5.2

c) Let A be a 3 x 3 matrix, what are $\mathbf{a_1}$, $\mathbf{a_2}$, and $\mathbf{a_3}$ in relation to matrix A? If $\mathbf{a_1} + 2\mathbf{a_2} = \mathbf{a_3}$, then how many solutions will the system $A\mathbf{x} = \mathbf{0}$ have? Explain. Is A invertable? Explain.

9) Let

$$A = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}, B = \begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}, \text{and } C = \begin{pmatrix} 3 & 7 \\ -2 & 5 \end{pmatrix}$$

Solve $AX + B = X + 2C$ for matrix X.

10) Given matrix A

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 5 & 3 & 1 \end{pmatrix}$$

a) Find $\det(A)$ by co-factors.

b) Find $\det(A)$ by elimination.

c) Does A have an inverse? Explain.

$$A = \begin{matrix} 1 & 2 & 3 \\ & & \\ 2 & 0 & 1 \\ 5 & 3 & 1 \end{matrix}$$

8

11) Given matrix A

$$A = \begin{pmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{pmatrix}$$

What conditions must the scalars a, b, and c satisfy for A to be singular?