Name:

Math 511 - Exam 2

1) For the set of vectors in $\mathbb{R}^2$ define addition normally but scalar multiplication by $\alpha \boldsymbol{x} = [\, x_1, \, \alpha x_2]^T$. Does this form a vector space? Explain. (Note: Axioms are given on the last page of the exam)

2a) Is the set of all polynomials $p(x)$ in P_4 such that $p(0) = 0$ (these are the polynomials whose graph passes through the origin) a subspace of P_4? Explain.

2b) Does the set of all 2 x 2 matrices A such that $a_{22} = 1$ form a subspace of $\mathbb{R}^{2\times 2}$? Explain.

3) Let $\boldsymbol{x}_1 = [1,\, 0,\, -1]^T$, $\boldsymbol{x}_2 = [2,\, 4,\, 0]^T$, and $\boldsymbol{x}_3 = [0,\, 2,\, 1]^T$. Are the vectors linearly independent? Prove your answer.

4) Are $\{(1), (1+x), (x+x^2), (x^2+1)\}$ linearly independent in P_3? Prove your answer.

5) Consider the vectors $\boldsymbol{x}_1 = [1, 2, 1]^T$, $\boldsymbol{x}_2 = [2, 5, 0]^T$, $\boldsymbol{x}_3 = [1, 3, -1]^T$, and $\boldsymbol{x}_4 = [3, 7, 1]^T$. What is the dimension for the Span of the vectors? Pare down and/or extend the vectors to make a basis for $\mathbb{R}^3$.

6) For P_2 with bases $B_1 = \{1-x, 1+2x\}$ and $B_2 = \{1-3x, 2+7x\}$, find the matrix, and call it S, representing the change of base from B_2 to B_1. Find the transition matrix, and call it T, representing the change of base from B_1 to B_2. DO NOT MULTIPLY THE MATRICES OR FIND ANY INVERSES FOR YOUR ANSWERS. Just leave them as a product of matrices and their inverses.

7) Determine if $L([x_1, x_2]^T) = [x_1, x_1, x_1 + x_2^2]^T$ from $\mathbb{R}^2$ to $\mathbb{R}^3$ is a linear operator.

8) Let A be a 3×4 matrix and U is the reduced row echelon form of A. If ...

$$U = \begin{pmatrix} 1 & -2 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

a) determine the rank(A), the nullity of A, and the dependency equations.

b) if $\boldsymbol{a}_1 = [1,2,3]^T$, $\boldsymbol{a}_2 = [-2,-4,-6]^T$, $\boldsymbol{a}_3 = [-1,1,-2]^T$, and $\boldsymbol{a}_4 = [-1,4,-1]^T$ then write the column space of A as a Span.

c) write the row space of A as a Span.

d) write N(A) as a Span.

9) Determine the kernel and range of the linear operator $L(\boldsymbol{p}) = x\boldsymbol{p}' + \boldsymbol{p}''$ on P_3.

10) For the linear operator $L(\boldsymbol{x}) = [\, x_1, x_2, x_1 + 2x_2 \,]^T$ from $\mathbb{R}^2$ into $\mathbb{R}^3$ find the standard linear operator matrix, A_E.

11) For the linear operator $L(\boldsymbol{p}) = x\boldsymbol{p}' + \boldsymbol{p}''$ on P_3 ...

... with standard basis $E = [(1), (x), (x^2)]$ and basis $B = [(1), (1+x), (x+x^2)]$

a) Find the matrix representation of L with respect to the standard basis, and call it A_E.

b) Find the matrix representation of L with respect to basis B, and call it A_B.

Vector Space Axioms

Let V be a set on which the closed operations of addition and scalar multiplication are defined. By this we mean that ...

C1) If $\boldsymbol{x} \epsilon V$ and α is a scalar, then $\alpha \boldsymbol{x} \epsilon V$.

C2) If $\boldsymbol{x}, \boldsymbol{y} \epsilon V$, then $\boldsymbol{x} + \boldsymbol{y} \epsilon V$.

The set V, together with the operations of addition and scalar multiplication, is said to form a vector space if the following axioms are satisfied:

A1) $\boldsymbol{x} + \boldsymbol{y} = \boldsymbol{y} + \boldsymbol{x}$

A2) $(\boldsymbol{x} + \boldsymbol{y}) + \boldsymbol{z} = \boldsymbol{x} + (\boldsymbol{y} + \boldsymbol{z})$

A3) There exists element $\mathbf{0}$ such that $\boldsymbol{x} + \mathbf{0} = \boldsymbol{x}$

A4) There exists element $-\boldsymbol{x}$ such that $\boldsymbol{x} + (-\boldsymbol{x}) = \mathbf{0}$

A5) $\alpha(\boldsymbol{x} + \boldsymbol{y}) = \alpha \boldsymbol{x} + \alpha \boldsymbol{y}$

A6) $(\alpha + \beta)\boldsymbol{x} = \alpha \boldsymbol{x} + \beta \boldsymbol{x}$

A7) $(\alpha\beta)\boldsymbol{x} = \alpha(\beta \boldsymbol{x})$

A8) $1 \cdot \boldsymbol{x} = \boldsymbol{x}$