NAME:

MATH 511 - EXAM 3

1) For the pair of vectors $\boldsymbol{x} = (1, -2, 3)^T$ and $\boldsymbol{y} = (1, 1, 1)^T$, find the scalar projection $\boldsymbol{x}$ onto $\boldsymbol{y}$, the vector projection $\boldsymbol{p}$ of $\boldsymbol{x}$ onto $\boldsymbol{y}$, and verify that $(\boldsymbol{x} - \boldsymbol{p}) \perp \boldsymbol{p}$.

2) Let A be a 3x4 matrix. Considering A as a linear transform describe its domain and codomain. For its codomain, is it possible to have the vector $(3, 1, 2)^T$ in the null space of A^T and $(-1, 0, 1)^T$ in the column space of A? Explain.

3) For an experiment you collect the following (x, y)-data points: $\{(-1, 1), (0, 1), (1, 2), (2, 2)\}$. As was explained in the exam review ... setup the matrices and equation to solve the least-squares fit to the data by a linear polynomial $y = a\,x + b$. DO NOT solve it.

4) In $C[-1, 1]$, with inner product $< f, g > = \int_{-1}^{1} f(x)\; g(x)\; dx$, find the vector projection $f(x) = 1$ onto $g(x) = x^2$.

5) Given inner product space $\mathbb{R}^{2\times 2}$ with $\langle A,\, B\rangle = a_{11}b_{11} + a_{12}b_{12} + a_{21}b_{21} + a_{22}b_{22}$ find the angle in arccos form between given matrices C and D.

$$C = \begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix} \text{ and } D = \begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix}$$

6) Verify that the functions $f_1(x) = \left(1 \,/\, \sqrt{2}\right)$ and $f_2(x) = \sqrt{3/2}\, x$ form an orthonormal set in $C[-1, 1]$ with the inner product defined by $< f, g > = \int_{-1}^{1} f(x) g(x)\, dx$.

7) Determine the value of ...

$\frac{1}{\pi} \int_{-\pi}^{\pi} (3\sin(x) - 2\cos(x))\,(\cos(x) + 2\sin(x))\, dx.$

... knowing that $\cos(x)$ and $\sin(x)$ are orthonormal. DO NOT actually do the integration.

8) Use the Gram-Schmidt process to find the QR factorization ...

$$A = \begin{pmatrix} 1 & 4 & 1 \\ 2 & 3 & 2 \\ -2 & 2 & 1 \end{pmatrix}$$

9) Find the eigenvalues and their corresponding eigenvectors of the matrix

$$A = \begin{pmatrix} 3 & 2 \\ 0 & -2 \end{pmatrix}$$

10) Find the eigenvalues and their corresponding eigenvectors of the matrix

$$A = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$$

11) For the given matrix A, find the equation $AX = XD$ where D is a diagonal matrix. Can matrix A be written as $A = XDX^{-1}$? Explain.

$$A = \begin{pmatrix} 9 & -5 & 3 \\ 0 & 4 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$