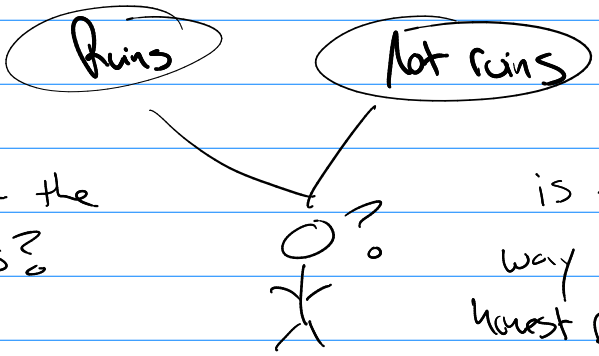
Math 321

Q5

1.2 #15



is the left fork the way to the ruins?
Honest person: yes
Dishonest person: no

is the right fork the way to the ruins?
Honest person: no
Dishonest person: yes

if I asked you if the left fork is the way to the ruins, would you say yes?

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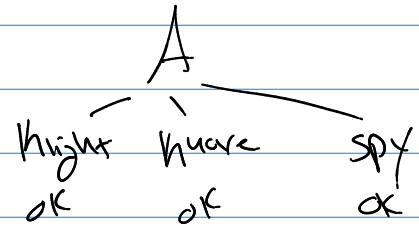
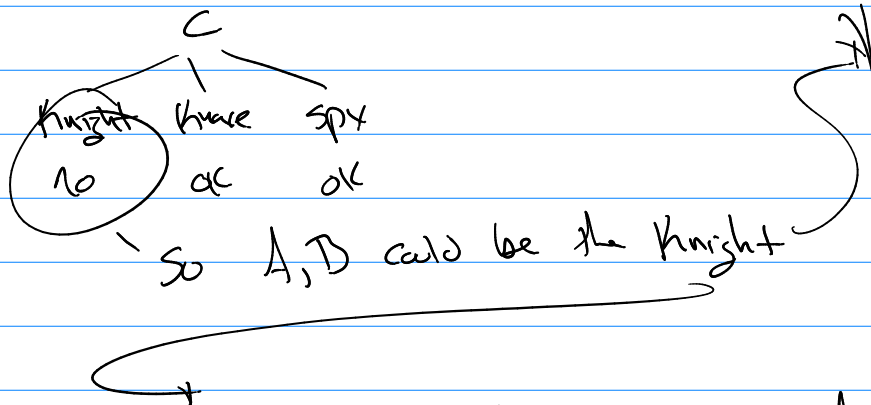
(27) A: "I am the Knight"
B: "A is telling the truth"
C: "I am the spy"





So A, B could be the Knight

A: Knight
→ B is spy (tells truth)
→ C is Knave

B is Knight then A is spy or Knave and they lie

So B can not be the Knight.

1.3 "Same?"

Discrete 2: we will study this property more.

Propositionally Equivalent

compound prop. #1 (vs) compound prop. #2

Terms

① **Tautology** - Compound proposition that is always true for truth values & its simple prop.

ex

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

② **Contradiction**: Compound prop that is always false

ex

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

③ **Contingency**: Sometimes T sometimes F

ex

p	q	$q \Rightarrow p$
T	T	T
T	F	T
F	T	F
F	F	T

Note:

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

logically equivalent prop #1 ⓧ prop #2

Def: $(\text{prop \#1}) \leftrightarrow (\text{prop \#2})$ if it is a tautology we say they are logically equiv.

Notation: $(\text{prop \#1}) \equiv (\text{prop \#2})$

ex $(p \to q)$ vs $(\neg p \vee q)$

p	q	$(p \to q)$	$\neg p$	$\neg p \vee q$	$(p \to q) \leftrightarrow (\neg p \vee q)$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

$(p \to q) \equiv (\neg p \vee q)$

Use? college Algebra $3(x+4) = 3x+12$

$x^2 + 2x = x(x+2)$

Prop. Logic (Equiv Laws)

$p \wedge T \equiv p$ $\quad p \vee F \equiv p$ Identity laws

$p \wedge F \equiv F$ $\quad p \vee T \equiv T$ domination laws

$p \vee \neg p \equiv T$ $\quad p \wedge \neg p \equiv F$ Negation laws

$p \vee p \equiv p$ $\quad p \wedge p \equiv p$ idempotent laws

$p \vee (p \wedge q) \equiv p$ $\quad p \wedge (p \vee q) \equiv p$ absorption laws

$\neg(\neg p) \equiv p$

$p \wedge q \equiv q \wedge p$ $\quad p \vee q \equiv q \vee p$ Commutative.

$(p \leftrightarrow q) \equiv (q \leftrightarrow p)$

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ assoc.

$(p \vee q) \vee r \equiv p \vee (q \vee r)$

Distributive ① De Morgan's $\quad \neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

② $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

etc