Math 321

Q's   1.1   15e

ex   □ is nec ($\rightarrow$) and ~~but~~ not suff ($\neg(\rightarrow)$) for △   ($\wedge$)

→ □ is nec. for △ $△ \rightarrow □$ ☆

→ □ is suff. for △ $□ \rightarrow △$

→ □ is not suff for △

it is not the case that, □ is suff for △ $\neg(□ \rightarrow △)$ ☆

$$(△ \rightarrow □) \wedge \neg(□ \rightarrow △)$$

15e for hiking, it is nec but not suff that $(\neg \text{ripe} \wedge \neg \text{bears})$

$$\left[\text{hiking} \rightarrow \underline{(\neg \text{ripe} \wedge \neg \text{bears})}\right] \wedge \neg\left[(\neg \text{ripe} \wedge \neg \text{bears}) \rightarrow \text{hiking}\right]$$

hiking	ripe	bears	$\neg$ripe $\wedge$ $\neg$bears	hike $\rightarrow$ ($\neg$ripe $\wedge$ $\neg$bears)
T	T	T	F	F
T	T	F	F	F
T	F	T	F	F
T	F	F	T	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	T	T

Distribution: $\neg(p\wedge q) \equiv \neg p \vee \neg q$ $\quad$ $\neg(p\vee q) \equiv \neg p \wedge \neg q$

$$p \to (q\wedge r) \equiv (p\to q) \wedge (p \to r)$$
$$p \to (q\vee r) \equiv (p\to q) \vee (p \to r)$$

$$p \wedge (q\vee r) \equiv (p\wedge q) \vee (p\wedge r)$$
$$p \vee (q\wedge r) \equiv (p\vee q) \wedge (p\vee r)$$

$$(q\wedge r) \to p \equiv (q\to p) \vee (r\to p)$$
$$(q\vee r) \to p \equiv (q\to p) \wedge (r\to p)$$

☆ $(p\to q) \equiv (\neg p) \vee q$ disjunctive version of implies

☆ $(p\to q) \equiv (\neg q \to \neg p)$

☆ $\neg(p\to q) \equiv \neg(\neg p \vee q) \equiv \boxed{p \wedge \neg q}$

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logical equivalences $\square \equiv \triangle$

① show logically equiv.

a) truth table for $\square \leftrightarrow \triangle$ is tautology

b) ask when is $\square$ true?
when is $\triangle$ true?
same conditions? → logically equiv.

or

ask when false?

$$\neg(p \to q) \equiv p \land \neg q$$

Note:

p	q	$p \to q$	$\neg(p \to q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

When true?

① $\neg(p \to q)$

T when $(p \to q)$ is false

when p is T <u>and</u> q is F

② $p \land \neg q$ T when p is T, $\neg q$ is T

when p is T and q is F

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vs when false?

① $\neg(p \to q)$, F when $p \to q$ is true

when (i) $p \equiv T \land q \equiv T$

(ii) $p \equiv F \land q \equiv T$

(iii) $p \equiv F \land q \equiv F$

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use?

$$(s \to (\neg r \land \neg b)) \land \neg((\neg r \land \neg b) \to s)$$

$\neg(p \to q) \equiv p \land \neg q$

$$\equiv (s \to (\neg r \land \neg b)) \land ((\neg r \land \neg b) \land \neg s)$$

$$\equiv (\neg s \lor (\neg r \land \neg b)) \land \neg r \land \neg b \land \neg s$$

$$\equiv (\neg s \lor \neg r) \land (\neg s \lor \neg b) \land \neg r \land \neg b \land \neg s$$

$$\equiv$$

Note

Proposition: "element e has property P"

predicate

Safe: "hiking is safe"

element

predicate

propositional function

propositional variable

pred(X): "X has property 'pred'"

predicate

X is from a set called the universe of discourse.

ex Safe(x): "X is safe"

UD: X is from the set of stuff you can do.

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n-ary predicate

$P(e_1, e_2, .., e_n)$: "$e_1, e_2, .., e_n$ have predicate P"

ex pos(x,y,z): "$x^2 + y^2 + z^2 > 0$"

U.D. for x, y, and z are all integers

turning prop function into a proposition?

ex Safe(x) : "x is safe" U.D of x is actions.

1st evaluation: Safe(doing math) : "doing math is safe"

Quantification
- 2nd all x of U.D. has predicate P (Universal)
- 3rd some x of U.D. has predicate P (Existential)