Math 321

bicondtional? $\square \leftrightarrow \triangle$

$\square$	$\triangle$	$\square \leftrightarrow \triangle$
T	T	T
T	F	F
F	T	F
F	F	T

Q5 1.3 # 17 $\neg(p \leftrightarrow q)$ vs $p \leftrightarrow \neg q$

logically equiv?

① truth table: p | q | $[\neg(p \leftrightarrow q)] \leftrightarrow (p \leftrightarrow \neg q)$

$\rightarrow$ all T's

$\rightarrow$ logically equiv.

② Argument? $\neg(p \leftrightarrow q)$ vs $(p \leftrightarrow \neg q)$

When True?

when is $(p \leftrightarrow \neg q)$ true? when $p, \neg q$ have same truth values

So this is when p, q have opposite truth values

$p \equiv T \quad \neg q \equiv T$

$p \equiv T \quad q \equiv F$

or $p \equiv F \quad \neg q \equiv F$

$p \equiv F \quad q \equiv T$

when is $\neg(p \leftrightarrow q)$ true?

So $p \leftrightarrow q$ is false, So p, q have opp. values

1.4/1.5 n-ary propositional function.

$P(e_1, e_2, \ldots, e_n)$: "$(e_1, e_2, \ldots, e_n)$ have predicate P"

UD for each variable

Love(□, △) : "□ loves △"

U.D for □ is people
U.D for △ is people

Love(Mark, Mark) : "Mark loves Mark"

Making propositions by...

① Evaluation

② Quantification — Universal ∀

③ Quantification — Existential ∃

$\forall x P(x)$: "for all x in U.D P(x) is True"

$\exists x P(x)$: "for [some x in U.D.] P(x) is True"

(some x in U.D. — one or more)

& U.D. is finite $o_1, o_2, o_3, \ldots, o_N$

$\forall x P(x) \equiv P(o_1) \wedge P(o_2) \wedge P(o_3) \wedge \ldots \wedge P(o_N)$

$\exists x P(x) \equiv P(o_1) \underline{\vee} P(o_2) \underline{\vee} P(o_3) \underline{\vee} \ldots \underline{\vee} P(o_N)$

Negation:

$\neg \forall x P(x) \equiv \exists x \neg P(x)$

$\neg \exists x P(x) \equiv \forall x \neg P(x)$

More than one predicate.

$P_1(x)$, $P_2(x)$

U.D. of x's are of the same set

(Ex) Tall(s): "s is taller than 6ft"
Hate(q): "q hates math"

→ U.D. for s, q are both students in room.

everyone is taller than 6ft $\forall a \, Tall(a)$

no one is taller than 6ft $\forall b \, \neg Tall(b)$
$\equiv \neg \exists b \, Tall(b)$

Note: $\forall a (Hate(a) \rightarrow Tall(a))$

$\forall a (Hate(a) \wedge Tall(a))$

$\exists a (Hate(a) \rightarrow Tall(a))$ —

$\exists a (Hate(a) \wedge Tall(a))$

N-ary predicates and Quantification → Nested Quantifiers

(Ex) Love(B, D): "B loves D"
B UD is people D UD is food

all people love all foods

$\forall p \forall f \text{ Loves}(p,f)$ — order?

$\forall p \forall f \text{ Loves}(p,f) \equiv \forall f \forall p \text{ Loves}(p,f)$

$\exists p \forall f \text{ Loves}(p,f) \quad \forall f \exists p \text{ Loves}(p,f)$