Math 321

Q5 1.9 (21d) "No two different people have the same grandma."

→ $sg(a,b)$: "a has same grandma as b"

UD & a,b is ... (UD = {Mark, Mark's brother})

$\forall a \forall b ((a \neq b) \wedge \neg sg(a,b))$ given this UD this is false

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1.5 1-ary predicates $P(o_1, o_2, .., o_n)$: "$(o_1, o_2, .., o_n)$ has predicate P"

UD & $o_1, o_2, .. o_n$ is ~

Binding (Make it a proposition)

① evaluation

② ③ } quantification $\forall$ $\exists$

Issues: #1 Scope $\forall n (sg(n, Mark) \wedge \exists b (b \neq n))$

n's scope

b's scope

$\forall x (P_1(x) \rightarrow P_2(x))$ is $\forall x P_1(x) \rightarrow \forall x P_2(x)$

x's scope

x's scope makes the x's different!

Better notation for right example $(\forall x P_1(x) \to \forall x P_2(x))$

use -- if $\forall p P_1(p) \to \forall h P_2(h)$

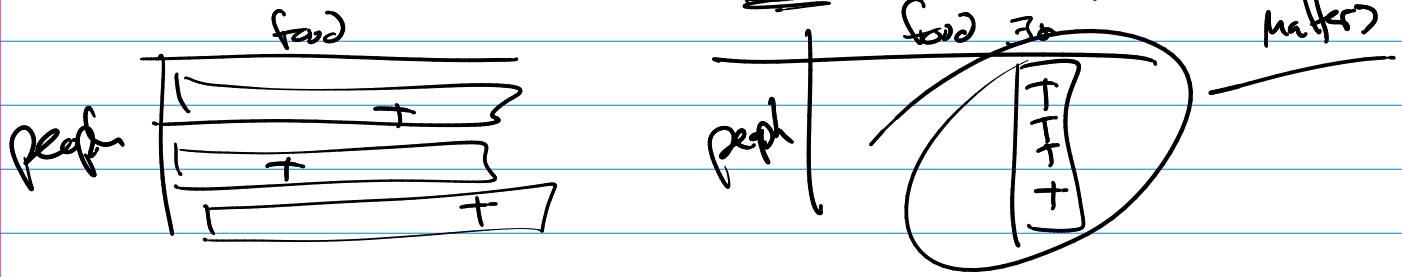
#2 order

$\forall x \forall y P(x,y) \equiv \forall y \forall x P(x,y)$

$\exists x \exists y P(x,y) \equiv \exists y \exists x P(x,y)$

→ same quantifier → order doesn't matter.

$\forall x \exists y P(x,y) \not\equiv \exists y \forall x P(x,y)$ order matters



#3 Negation

$\neg \forall p \exists q \forall r \; P(p,q,r)$

$\equiv \exists p \forall q \exists r \neg P(p,q,r)$

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ex $\neg \forall a \forall b (a \neq b \wedge \neg Sg(a,b))$

$\equiv \exists a \exists b \neg (a \neq b \wedge \neg Sg(a,b))$

$\equiv \exists a \exists b (a = b \vee Sg(a,b))$

1.6 Useful tautologies

Arguments | Seq. of statements (premises) that end in a conclusion.

If (premise 1 ∧ premise 2 ∧ ... ∧ premise n) → conclusion

(left) → (right)

left	right	left → right
T	T	T
T	F	F
F	T	T
F	F	T

Valid argument is to say it is a tautology

Argument form: Argument using prop. variables

ex $((p \lor q) \land (r \to s) \land (\neg r)) \to (s \leftrightarrow p)$

ex $((p \to q) \land (p)) \to (q)$

tautology? ✓ Valid! ✓ not a tautology? ✓ not Valid! ✓

Rules of Inference (Valid argument forms that we use a lot)

$[(p \to q) \land p] \to q$

Modus Ponens

$$\frac{p \to q \quad p}{\therefore q}$$