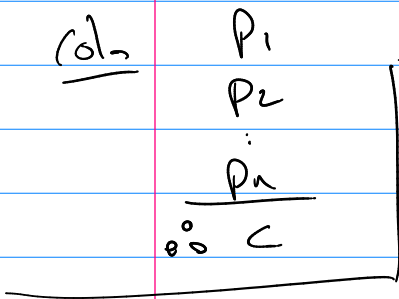


# Math 321

Valid Arguments :  $\underbrace{(p_1 \wedge p_2 \wedge \dots \wedge p_n)}_{\text{Argument}} \rightarrow c$  is a tautology  
(Valid)



Rules of Inference

Modus ponens  
(affirm the hypothesis)

$$\frac{p \rightarrow q}{\therefore q}$$

Verify:

| $p$ | $q$ | $p \rightarrow q$ | $(p \rightarrow q) \wedge p$ | $[(p \rightarrow q) \wedge p] \rightarrow q$ |
|-----|-----|-------------------|------------------------------|--|
| T   | T   | T                 | T                            | T  |
| T   | F   | F                 | F                            | T  |
| F   | T   | T                 | F                            | T  |
| F   | F   | T                 | F                            | T  |

(c)  $\frac{p \rightarrow q}{\neg q} \quad ((p \rightarrow q) \wedge (\neg q)) \rightarrow \neg p$

$\therefore \neg p$

modus tollens

(denying the conclusion)

$$\begin{array}{l}
 p \rightarrow q \\
 q \rightarrow r \\
 \hline
 \therefore p \rightarrow r
 \end{array}$$

hypothetical syllogism

$$\begin{array}{l}
 p \vee q \\
 \neg p \\
 \hline
 \therefore q
 \end{array}$$

disjunctive syllogism

$$\begin{array}{l}
 p \\
 \hline
 \therefore p \vee q
 \end{array}$$

addition

$$\begin{array}{l}
 p \wedge q \\
 \hline
 \therefore q
 \end{array}$$

simplification

conjunction:

$$\begin{array}{l}
 p \\
 q \\
 \hline
 \therefore p \wedge q
 \end{array}$$

$$\begin{array}{l}
 p \vee q \\
 \neg p \vee r \\
 \hline
 \therefore q \vee r
 \end{array}$$

resolution

Quantifiers

$$\begin{array}{l}
 \forall x P(x) \\
 \hline
 \therefore P(c) \text{ for any } c
 \end{array}$$

Universal  
instantiation

$$\begin{array}{l}
 \exists x P(x) \\
 \hline
 \therefore P(c) \text{ for a specific } c
 \end{array}$$

Existential  
instantiation

$$\begin{array}{l}
 P(c) \text{ for all } c \text{ in U.D.} \\
 \hline
 \therefore \forall x P(x)
 \end{array}$$

Universal  
Generalization

$P(c)$  for some  $c$

$\therefore \exists x P(x)$

Existential Generalization

(ex) "no man is an island"  
-----  
 $\therefore$  "Mark is not an island"

$\rightarrow \exists x (Man(x) \wedge Island(x))$   
 $\equiv \forall x \neg (Man(x) \wedge Island(x))$

$\equiv \forall x (\neg Man(x) \vee \neg Island(x))$

$\equiv \forall x (Man(x) \rightarrow \neg Island(x))$

U.D.  
Woman

$Man(x)$ : "x is a man"

$Island(x)$ : "x is an island"

"no man is an island"

$\forall x (Man(x) \rightarrow \neg Island(x))$

conclude: "if Mark is a man  
then mark is not an island" Univ Inst.

$(Man(Mark) \rightarrow \neg Island(Mark))$

"Mark is a man"

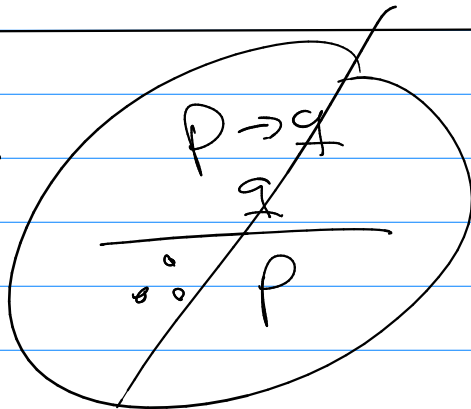
Enthymeme

$Man(Mark)$  ← unstated  
premise

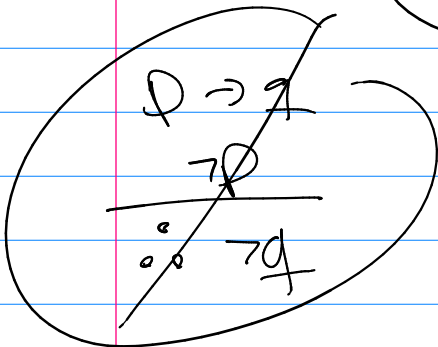
$\therefore$  "Mark is not an island"

$\therefore \neg Island(Mark)$

Fallacies



Fallacy of affirming  
the conclusion



Fallacy of  
denying the hyp.

1.7/18

use of valid arguments:

Proofs

→ Statement that the "think" may be true.

Conjectures

all reasoning (Axiomatic Method)

