Math 321

Valid Arguments: $\underbrace{(p_1 \wedge p_2 \wedge \ldots \wedge p_n) \to c}_{\text{Argument}}$ is a tautology (Valid)

eg. p_1
p_2
$\vdots$
p_n
$\therefore\ c$

Rules of Inference

Modus ponens
(affirm the hypothesis)

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore q \end{array}$$

Verify:

p	q	$p \to q$	$(p \to q) \wedge p$	$[(p \to q) \wedge p] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

ex.

$$\begin{array}{l} p \to q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

$((p \to q) \wedge (\neg q)) \to \neg p$

Modus tollens
(denying the conclusion)

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

hypothetical syllogism

$$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

disjunctive syllogism

$$\begin{array}{l} p \\ \hline \therefore p \vee q \end{array}$$

addition

$$\begin{array}{l} p \wedge q \\ \hline \therefore q \end{array}$$

Simplification

Conjunction:

$$\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array}$$

$$\begin{array}{l} p \vee q \\ \neg p \vee r \\ \hline \therefore q \vee r \end{array}$$

resolution

Quantifiers

$$\begin{array}{l} \forall x\, P(x) \\ \hline \therefore P(c) \end{array}$$

for any c

Universal instantiation

$$\begin{array}{l} \exists x\, P(x) \\ \hline \therefore P(c) \end{array}$$

for a specific c

Existential instantiation

$$\begin{array}{l} P(c) \text{ for all } c \text{ in U.D.} \\ \hline \therefore \forall x\, P(x) \end{array}$$

Universal Generalization

$P(c)$ for some c

$\therefore \exists x\, P(x)$

Existential Generalization

(a) "No man is an island"

$\therefore$ "Mark is not an island"

$\neg \exists x\, (Man(x) \land Island(x))$

$\equiv \forall x\, \neg(Man(x) \land Island(x))$

$\equiv \forall x\, (\neg Man(x) \lor \neg Island(x))$

$\equiv \forall x\, (Man(x) \rightarrow \neg Island(x))$

U.D. (humans)

$Man(x)$: "x is a man"

$Island(x)$: "x is an island"

"No man is an island" $\qquad \forall x\, (Man(x) \rightarrow \neg Island(x))$

Conclusion: "If Mark is a man then Mark is not an island" $\qquad$ Univ Inst. $\qquad (Man(Mark) \rightarrow \neg Island(Mark))$

"Mark is a man" $\qquad$ Enthymeme $\qquad Man(Mark) \leftarrow$ unstated premise

---

$\therefore$ Mark is not an island $\qquad \therefore \neg Island(Mark)$

Fallacies

$$p \rightarrow q$$
$$q$$
$$\therefore p$$

Fallacy of affirming the conclusion

$$p \rightarrow q$$
$$\neg p$$
$$\therefore \neg q$$

fallacy of denying the hyp.

1.7/1.8 use of valid arguments: Proofs

→ Statement that they "think" may be true. | Conjectures

all reasoning (Axiomatic Method)

proved statements: (facts, theorems, etc)

proof → Make new stuff

just "are" → undefined terms / Postulates or axioms → true statement