Math 321

Q's (23) 1.6 $\exists x P(x) \wedge \exists x Q(x)$

argument:

(1) $\exists x P(x) \vee \exists x Q(x)$ ← incorrect (type)

(2) $\exists x P(x)$ ← Simplification (incorrect use)

(3) $P(c)$ ← incorrect (needs "for some c")

(4) $\exists x Q(x)$ ← incorrect simplification

(5) $Q(c_2)$

(6) $P(c) \wedge Q(c)$ ← correct but c's are different

$\therefore \exists x (P(x) \wedge Q(x))$ incorrect b/c

Valid arguments application: show interesting statements are true.

proof

terms: Conjecture: statement we think may be true

show true (proof) it becomes a --

1. fact or result
2. lemma
3. theorem
4. corollary

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ex: show $\sqrt{2}$ is irrational (eventually)

Step 1: what does the conjecture say? what form is it?

form of statements:

type #1 Conjecture of the form "$a \to b$"

Consider

a	b	a → b
T	T	T
T	F	F
F	T	T
F	F	T

Special cases

① Vacuous proof
show $a \equiv F$

② trivial proof
show $b \equiv T$

to prove $a \to b$

① Direct Proof

assume $a \equiv T$ now show b must be T.

example: prove if a is even, then a^2 is even

Proof: Direct proof

assume a is even

by def $a = 2 \cdot K$, K is an integer

$$(a)^2 = (2K)^2$$

$$a^2 = 4K^2 = 2\underbrace{(2K^2)}_{\text{integer}}$$

So a^2 is even

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try a^2 is even $\to$ a is even

pf (try direct)

assume a^2 is even, so $a^2 = 2 \cdot k$, k an int.

$\sqrt{a^2} = \sqrt{2k}$

$a = ???$

goal? $a = 2(\text{int})$

$2(k) = 2(\quad)$

consider $(a \to b) \equiv (\neg b \to \neg a)$

contrapositive of $a \to b$

Indirect Proofs

① Prove the contrapositive of $(a \to b) \equiv (\neg b \to \neg a)$

ex $(a^2$ is even $\to a$ is even$)$

use contrapositive $\equiv (a$ is odd $\to a^2$ is odd$)$

pf (direct)

assume a is odd. So $a = 2k+1$, k is int

So $a^2 = (2k+1)^2 = 4k^2 + 4k + 1$

So $a^2 = 2(2k^2 + 2k) + 1$ so a^2 is odd

($2k^2 + 2k$: int)

Show: a is odd $\to a^2$ is odd is true

$a \cdot a = a^2$

∎

and a^2 is even $\to a$ is even