

Math 321

Q5

(23) 1.6

$$\exists x P(x) \wedge \exists x Q(x)$$

argument

- ① $\exists x P(x) \vee \exists x Q(x)$ incorrect (type)
 - ② $\exists x P(x)$
 - ③ $P(c)$ Simplification (incorrect use)
 - ④ $\exists x Q(x)$ incorrect (needs "for some c")
 - ⑤ $Q(c)$ incorrect simplification
 - ⑥ $P(c) \wedge Q(c)$ correct but c's are different
- $\therefore \exists x (P(x) \wedge Q(x))$ incorrect b/c

Valid arguments

application: show interesting statements are true.

proof

terms: Conjecture: statement we think may be true

show true (proof) it becomes a --

- ① fact or result
- ② lemma
- ③ theorem
- ④ corollary

ex show $\sqrt{2}$ is irrational (eventually)

Step 1

what does the conjecture say?
what form is it?

form of statements:

type #1

conjecture of

" $a \rightarrow b$ "

consider

a	b	$a \rightarrow b$
T	T	T
T	F	F
F	T	T
F	F	T

Special cases

① vacuous proof
show $a \equiv F$

② trivial proof
show $b \equiv T$

to prove $a \rightarrow b$

① Direct Proof

assume $a \equiv T$ now show b must be T .

example:

prove if a is even, then a^2 is even

Proof: Direct proof

assume a is even

by def $a = 2 \cdot k$, k is an integer

$$(a)^2 = (2k)^2$$

$$a^2 = 4k^2 = 2 \left(\underbrace{2k^2}_{\text{integer}} \right)$$

So a^2 is even



try

a^2 is even $\rightarrow a$ is even

PF

(try direct)

assume a is even, so

$$a = 2k, \quad k \text{ an int.}$$

k an int.

$$\sqrt{2k} = 2(\quad)$$

$$\sqrt{a^2} = \sqrt{2k}$$

$$a = \dots$$

goal? $a = 2(\text{int})$

consider

$$(a \rightarrow b) \equiv (\neg b \rightarrow \neg a)$$

$$(\neg b \rightarrow \neg a)$$

contrapositive of $a \rightarrow b$

Indirect Proofs

① Prove the contrapositive of $(a \rightarrow b) \equiv (\neg b \rightarrow \neg a)$

ex) $(a^2 \text{ is even} \rightarrow a \text{ is even})$

use contrapositive $\equiv (a \text{ is odd} \rightarrow a^2 \text{ is odd})$

PF (direct)

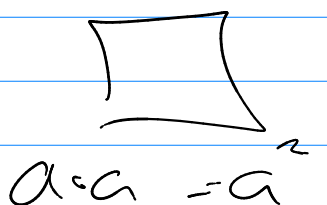
assume a is odd, so $a = 2k + 1, k \text{ is int}$

$$\text{So } a^2 = (2k + 1)^2 = 4k^2 + 4k + 1$$

$$\text{So } a^2 = 2(\underbrace{2k^2 + 2k}_{\text{int}}) + 1 \quad \text{so } a^2 \text{ is odd}$$

Show: $a \text{ is odd} \rightarrow a^2 \text{ is odd}$ is true

and $a^2 \text{ is even} \rightarrow a \text{ is even}$



$$a = a = a^2$$

