

# Math 321

## Q's Exm Prep

- Know?
- ① Operations
  - ② Logical Equiv.
  - ③ rules & inference

Spread out time to study  $\rightarrow$  Do not cram

1.6 #9a off: "I take day off"

off(x): "I take x off"

rain(x):

snow(x):

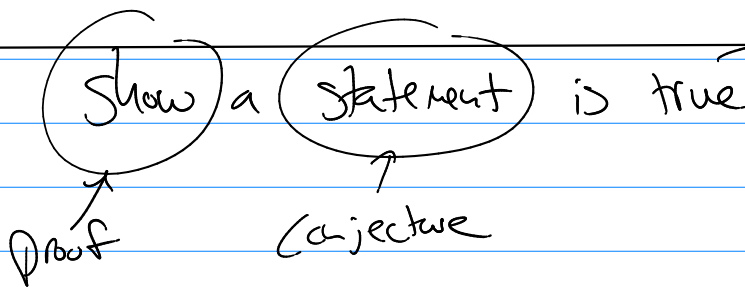
$\forall d (off(d) \rightarrow (rain(d) \vee snow(d)))$

U.D. for all 13 days

$off(\text{tues}) \rightarrow (rain(\text{tues}) \vee snow(\text{tues}))$

Sunny  $\rightarrow \neg rain(\text{tues}) \wedge \neg snow(\text{tues}) \equiv \neg (rain(\text{tues}) \vee snow(\text{tues}))$

## Proofs



- once you do this we call it a
- ① fact / result
  - ② lemma
  - ③ theorem
  - ④ corollary

## Types & statements

#1  $(a \rightarrow b)$

# techniques to prove $(a \rightarrow b)$

- ① trivial proofs
- ② vacuous proofs
- ③ Direct Proof

Valid Argument (Proof)

Assume  $a \equiv T$ , Show  $b \equiv T$

ex show  $a$  even  $\rightarrow \bar{a}$  even (last class)

## Indirect Proofs

- ④ Contradiction

$$\text{b/c } (a \rightarrow b) \equiv \boxed{(\neg b \rightarrow \neg a)} \leftarrow \text{prove this}$$

- ⑤ Proof by contradiction

want? (statement)  $\equiv T$

Same as  $\neg(\text{statement}) \equiv \neg T \equiv F$

so  $\neg(a \rightarrow b) \equiv F$  Show

①  $\neg \neg b \equiv F$

ex Show:  $(\sqrt{2})$  is irrational

①  $\sqrt{2}$  is irrational  
A

PF assume  $\neg(\sqrt{2}$  is irrational)

so  $\sqrt{2}$  is rational

so  $\sqrt{2} = \frac{a}{b}$   $\left\{ \begin{array}{l} \text{① } a, b \text{ are integers} \\ \text{② } b \neq 0 \end{array} \right.$

$\rightarrow$  ③  $a, b$  have no common factors

So  $\sqrt{2} = \frac{a}{b}$ , then  $\sqrt{2} \cdot \sqrt{2} = \frac{a}{b} \cdot \frac{a}{b}$

$$\text{so } 2 = \frac{a^2}{b^2} \rightarrow \text{b/c } \text{②}$$

$$2b^2 = a^2$$

$a^2$  is even  
b/c  $b$  is an int.

last class: lemma:  $a^2$  is even  $\rightarrow a$  is even

(PF) (used contrapositive)

do in  
the exam

So by lemma  $a$  is even ( &  $\sqrt{2} = \frac{a}{b}$  )

by def let  $a = 2 \cdot k$ ,  $k$  is an int.

above:  $2 = \frac{a^2}{b^2}$  become  $2 = \frac{(2k)^2}{b^2}$

$$\rightarrow 2 = \frac{4k^2}{b^2} \rightarrow b^2 = 2k^2 \rightarrow \underline{b^2 \text{ is even}}$$

by lemma  $b$  is even

together ( $a$  is even)  $\wedge$  ( $b$  is even)

(but) by (3) above  $a, b$  have no common factors

$\equiv \text{F}$

what happened ...?

(1) lemma  $a^2$  even  $\rightarrow a$  even

(2) showed  $\neg (\sqrt{2} \text{ is irrational}) \equiv \text{F}$

$\therefore \sqrt{2}$  is irrational  $\equiv \text{T}$