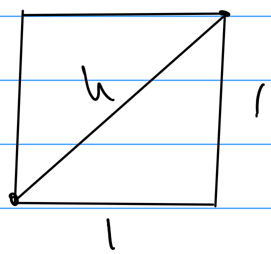
Math 321

Proofs Statements of type "$a \to b$" or just "b"

tech
1. Trivial
2. Vacuous
3. Direct Proof "assume a, show b"

Indirect proofs
4. Contraposition prove: $\neg b \to \neg a$
5. Contradiction prove $\neg(\text{Statement}) \equiv F$



$1^2 + 1^2 = h^2$ $\quad h^2 = 2$

$h = \sqrt{2}$

rational $\frac{a}{b} = a\left(\frac{1}{b}\right)$

Other statements

tautology

1. statement: "$a \leftrightarrow b$" proved

Shown $a \equiv b$

tech #1 $a \equiv S_1 \equiv S_2 \equiv \dots \equiv S_n \equiv b$

issues: $(x = 1) \overset{?}{\equiv} (x^2 = 1^2)$ $\quad x^2 = 1$

soln's $\qquad$ soln's $\qquad x^2 - 1 = 0$

$(x+1)(x-1) = 0$

$x = -1$ $\quad$ $x = 1$

tech #2 $(a \leftrightarrow b) \equiv (a \to b) \wedge (b \to a)$

case #1 $(a \to b)$

case #2 $(b \to a)$

② Statement: $(p_1 \vee p_2 \vee p_3 \vee \dots \vee p_n) \to q$

$$\equiv \underbrace{(p_1 \to q)}_{\text{case \#1}} \wedge \underbrace{(p_2 \to q)}_{\text{case \#2}} \wedge \dots \wedge \underbrace{(p_n \to q)}_{\text{case \#n}}$$

proof by cases

if cases are finite call it proof by exhaustion.

$$\boxed{(p \vee q) \to r \underset{\leftrightarrow}{\equiv} (p \to r) \wedge (q \to r)}$$

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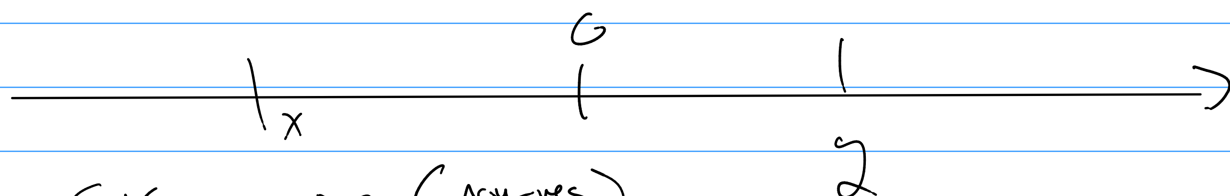
ex prove: #7 p.108 $\boxed{|x+y| \le |x| + |y|}$



$$|x| = \begin{cases} x & x \ge 0 \text{ (non-neg)} \\ -x & x < 0 \text{ (neg)} \end{cases}$$

cases:
① x, y both non-neg
② x, y both neg
③ x non-neg y neg
④ x neg y non-neg

← without loss of generality just do ③

pf Case #1 x, y both non-neg then $|x| = x$ $|y| = y$

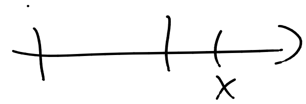
so $x+y$ is also non-neg $|x+y| = x+y$

$|x+y|$ vs $|x| + |y|$

$x+y$ vs $\overset{?}{\le}$ $(x+y)$ true!

Note: $\overset{\text{pos}}{x} + \overset{\text{neg}}{y}$ b/c x is pos, y is neg

(watch video)



y

$|y| > |x|$

Existence $\exists x P(x)$ → the c that make $P(c)$ true is the witness.

① find counter examples

Some one says $\forall x\, Q(x)$ you don't think this is true.

you find $\exists x \neg Q(x)$

c found is your counter example

② $\exists x P(x)$ → find your witness for $P(c)$

constructive proof: 1, 2, 3, $\boxed{4}$, 5, 6, 7, 8, $\boxed{9}$, 10, 11, ..

→ find the witness. (ex) square number beside cube number

Non-constructive proof — show $P(c)$ exists without being able to specifically name c.

(ex) there is a (irrational)$^{(\text{irrational})}$ is rational.

pf all real numbers are rational ⊕ irrational

Consider $\sqrt{2}^{\sqrt{2}}$ → case #1 is rational → then $\sqrt{2}^{\sqrt{2}}$ is witness.

→ case #2 is irrational

So $\left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = 2$ then $\left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}}$ is witness. ∎