Math 321

Sets

our 'new' toys

Representations
① list / roster
② set builder
③ Venn Diagrams
④ Membership tables

Sets to know
$\emptyset$, U
Numbers: $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Z}^+$, $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$

Cardinality how much is in my set?

$|S|$ = number of uniq. elements

if $|S| = n \in \mathbb{N}$ then call S **finite**

special: $|\emptyset| = 0$

ex $|\{\bigcirc, ☺, \triangle, 4\}| = 4$

$|\{\{\}, \{1,2,3\}\}| = 2$

??, $|\mathbb{N}|$ is not finite, call it **infinite**

$|\mathbb{Q}|$ infinite

$|\mathbb{R}|$ infinite

Comparing Sets

① if $|A| = |B|$ call A, B equivalent $A \sim B$

② $A = B$ mean the $(e \in A \Leftrightarrow e \in B)$

ex $\{1,1,2,3\} = \{1,2,3,3,3\}$

$\{1,2,3\} \neq \{1,2,3,4\}$

③ subset $A \subseteq B$ $\quad \forall e (e \in A \to e \in B)$

two special cases ⓘ for any set S, $\boxed{\emptyset \subseteq S}$

check $\forall e (e \in \emptyset \to e \in S)$

$\equiv \forall e (F \to e \in S) \equiv T$

ⓘⓘ for any set S, $\boxed{S \subseteq S}$

check $\forall e (e \in S \to e \in S) \equiv T$

Note: if you are asked to show $A \subseteq B$

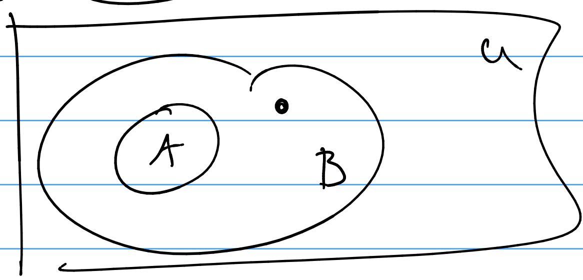
Need to prove $\forall e (\boxed{e \in A} \to \boxed{e \in B})$

do a $p \to q$ proof
(direct?, contrapositive?
contradiction?)

④ proper subset $A \subseteq B \wedge A \neq B$

$A \subset B$

book logic notation: $\forall x (x \in A \to x \in B) \wedge \exists x (x \in B \wedge x \notin A)$



Better? $\forall e_1 (e_1 \in A \to e_1 \in B) \wedge \exists e_2 (e_2 \in B \wedge e_2 \notin A)$

e e e f f f

Note: showing $A = B$ $\forall e (\underline{e \in A} \leftrightarrow \underline{e \in B})$

two techniques #1 $p \leftrightarrow q$ proof

#2 $(p \to q) \wedge (q \to p)$ $\boxed{(A \subseteq B) \wedge (B \subseteq A)}$

So far: Sets ← toys Comparisons → $A \sim B$, $A = B$, $A \subseteq B$, $A \subset B$

Note: know the notation

$A \subseteq B$

means → $\forall e (e \in A \to e \in B)$

↑ mean?

operations

two special operations

① cross product

$A_1 \times A_2 \times A_3$ → take an element from each set

(a_1, a_2, a_3)

$$A_1 \times A_2 \times .. \times A_n = \{ (e_1, e_2, .., e_n) \mid e_1 \in A_1 \wedge e_2 \in A_2 \wedge .. \wedge e_n \in A_n \}$$

ex $\{1, 2\} \times \{a, b, c\} = \{ (1,a), (1,b), (1,c), (2,a), (2,b), (2,c) \}$

Note: $|A_1 \times A_2 \times .. \times A_n| = |A_1| \, |A_2| \, .. \, |A_n|$

② **Power Set** = Set that collects all subsets of a set S

$$P(S) = \{ \text{every subset of } S \}$$

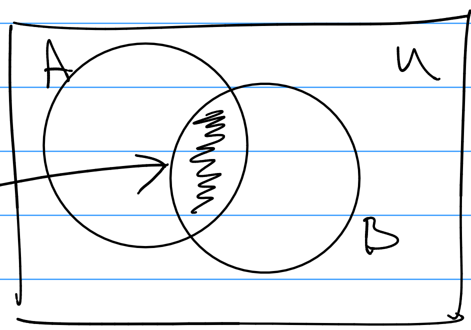
ex $P(\{1,2\}) = \{ \emptyset,$ — take 0

$\{1\}, \{2\},$ — take 1

$\{1,2\} \}$ — take 2

Note: $|P(S)| = 2^{|S|}$

Closed Ops



① Union $A \cup B = \{ e \mid e \in A \vee e \in B \}$

② Intersection $A \cap B = \{ e \mid e \in A \wedge e \in B \}$

③ Difference $A - B = \{ e \mid e \in A \wedge e \notin B \}$

④ Complement $\overline{A} = U - A = \{ e \mid e \notin A \}$