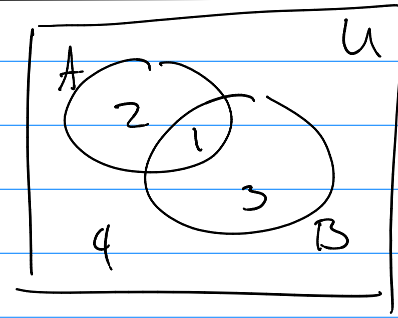
Math 321

Set Operations



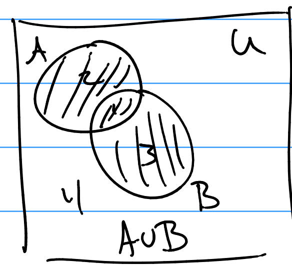
① Union $A \cup B = \{e \mid e \in A \vee e \in B\}$

② Intersection $A \cap B = \{e \mid e \in A \wedge e \in B\}$

③ Difference $A - B = \{e \mid e \in A \wedge e \notin B\}$
$= \{e \mid e \in A \wedge \neg(e \in B)\}$

④ Complement $\overline{A} = \{e \mid e \notin A\}$
$= \{e \mid \neg(e \in A)\}$

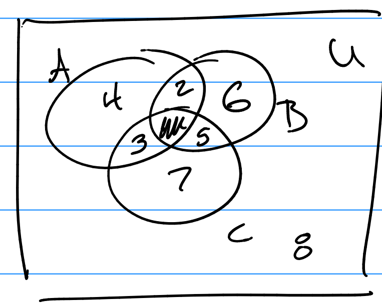
A	B	$A \cup B$
1	1	1
1	0	1
0	1	1
0	0	0



$A \cup B$

ex $A - \overline{(B \wedge C)}$ Venn?

A	B	C	$B \cap C$	$\overline{B \cap C}$	$A - \overline{B \cap C}$
1	1	1	1	0	1
1	1	0	0	1	0
1	0	1	0	1	0
1	0	0	0	1	0
0	1	1	1	0	0
0	1	0	0	1	0
0	0	1	0	1	0
0	0	0	0	1	0



A	B	$A - B$
1	1	0
1	0	1
0	1	0
0	0	0

Note: Laws

elem. alg.	$x + y = y + x$	comm. law
logic	$p \vee q \equiv q \vee p$	comm. law
Set theory	$A \cup B = B \cup A$	comm. law

laws $A \cap U = A \quad A \cup \emptyset = A$ identity laws

$A \cap \emptyset = \emptyset \quad A \cup U = U$ domination laws

2.2 rest laws

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

ex distributive

Show equal sets?

tech #1 set builder + logic

$\boxed{A \cap (B \cup C)} = \{e \mid e \in A \wedge e \in (B \cup C)\}$

$= \{e \mid \boxed{e \in A} \wedge (\boxed{e \in B} \vee \boxed{e \in C})\}$

use $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

$= \{e \mid (e \in A \wedge e \in B) \vee (e \in A \wedge e \in C)\}$

$= \{e \mid e \in (A \cap B) \vee e \in (A \cap C)\}$

$= \boxed{(A \cap B) \cup (A \cap C)}$

tech #2 use membership table

tech #3 prove if $(e \in \text{left}) \rightarrow (e \in \text{right})$

and if $(e \in \text{right}) \rightarrow (e \in \text{left})$

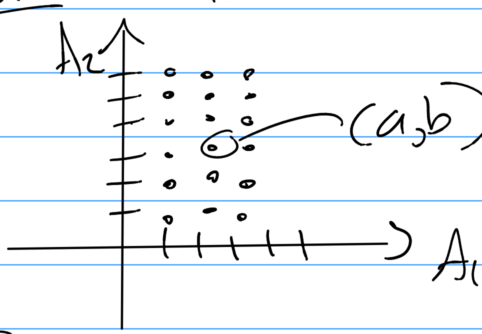
So left set = right set

ex $\overline{\overline{A}} = A$

Applications of Sets ?

Note: $A_1 \times A_2 = \{ (a,b) \mid a \in A_1 \wedge b \in A_2 \}$

set of all ordered pairs



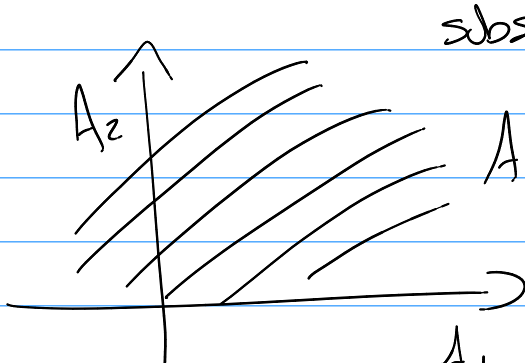
n sets

$$A_1 \times A_2 \times \dots \times A_n = \{ \underbrace{(a_1, a_2, \dots, a_n)}_{n\text{-tuples}} \mid \forall i \ a_i \in A_i \}$$

Relationship take some of the n-tuples.

① subset of $A_1 \times A_2 \times \dots \times A_n$ — n-ary relation.

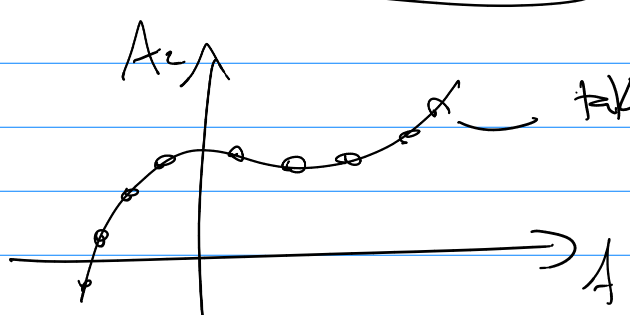
② $A_1 \times A_2$ study binary relations
subset of $A_1 \times A_2$



$A_1 \times A_2$ is all (a,b)

binary relation
↑
subset of

ex consider



take this subset

Study Functions

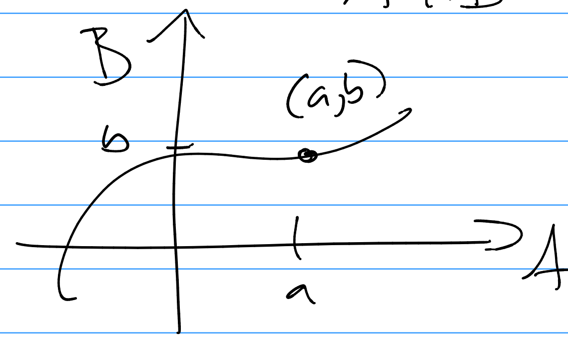
(they are 1^{st} binary relations)

subset of $A \times B$

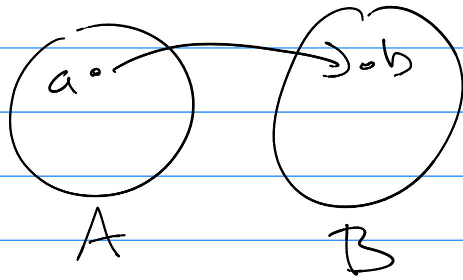
rule of relationship

Notation: $f : A \xrightarrow{f} B$

function name — Domain — Codomain



$f(a) = b$



a is preimage of b

b is the image of a

a is mapped by f to b

ex $f(x) = x^2 - 4$ ← rule

$\vdots$

$0 \to -4$

$1 \to -3$

$2 \to 0$

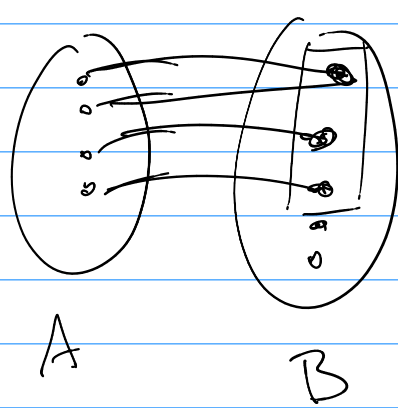
$3 \to 5$

$\vdots$

Def: a function assigns

1^{st} exactly one $b \in B$ to

2^{nd} every $a \in A$.

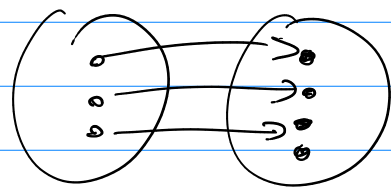


Range = Set of images

Note: Alg / Calculus (1 and 2) all $f: \mathbb{R} \to \mathbb{R}$

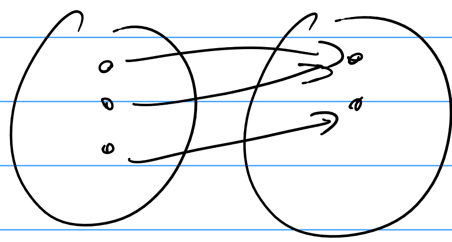
Special Functions (types)

① one-to-one function (injection)



preimages have their own uniq image.

② onto function (surjection)



codomain = range

③ bijection = one-to-one $\boxed{\text{and}}$ onto