Math 321

Q2's Exam 1 95 pts = 100%

2.1 #25 $P(A) \subseteq P(B)$ iff $A \subseteq B$

Pf case 1 $(A \subseteq B) \to (P(A) \subseteq P(B))$ ✓

case 2 $(P(A) \subseteq P(B)) \to (A \subseteq B)$ ✓

Note ① $S_1 \subseteq S_2$ mean?

any
$\forall e$ $e \in S_1 \to e \in S_2$ ← assume any throughout proof

② $P(S) = \{$ all of S's subsets $\}$

set of all of S's subsets

$= \{ \emptyset$ take nothing

$\{e_1\}, \{e_2\}, \ldots$ take 1 element of S

take 2 elements

⋮

$S \}$

③ $P(A) \subseteq P(B)$

says any element of P(A) is in P(B) as well

$e \in P(A) \to e \in P(B)$

I know e is a subset of A

So $P(A) \subseteq P(B)$ says any subset of A is a subset of B

as symbols $S \subseteq A \to S \subseteq B$

Case 1 $\quad A \subseteq B \to P(A) \subseteq P(B)$

Direct $\quad$ Assume $\boxed{A \subseteq B}$ & show $P(A) \subseteq P(B)$

but we know $P(A) \subseteq P(B)$ is simply

to show $\boxed{S \subseteq A} \to S \subseteq B$

$S \subseteq A$ and b/c $A \subseteq B \to S \subseteq B$ (by trans prop of subsets)

yes

Case 2 $\quad \left(P(A) \subseteq P(B)\right) \to (A \subseteq B)$

to do this just look a $P(A)$'s singletons subsets.

(Finish)

Applications of $f: A \to B$

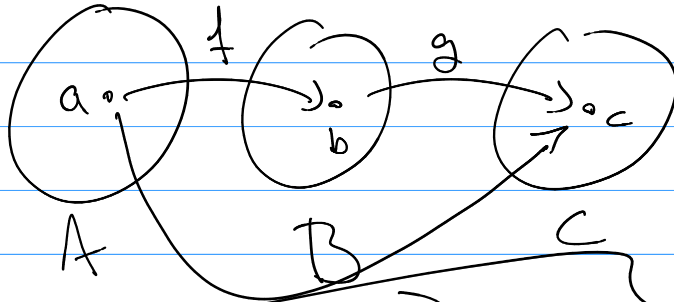
1st if (ex) $f: \mathbb{R} \to \mathbb{R}$ where you have known operations on domain and codomain

Define $(f \pm g)(x) = f(x) \pm g(x)$

$(fg)(x) = f(x) \cdot g(x)$

$(f/g)(x) = f(x)/g(x) \quad (g(x) \neq 0)$

$f: A \to B$ $\quad$ $g: B \to C$



$\boxed{g(f(a)) = c}$

Composition

$(g \circ f)(a) = c$

Identity = do nothing

op	element	
addition	0	b/c $a+0=a$
mult.	1	b/c $a \cdot 1 = a$
composition	$f(x)=x$	b/c $g(f(x)) = g(x)$, $f(g(x)) = g(x)$

Inverse undo concept

$$x + 5 = y$$

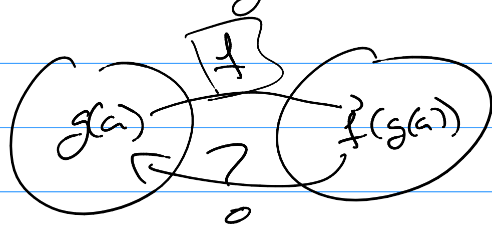
$$x + \boxed{5 + (-5)} = y + (-5)$$

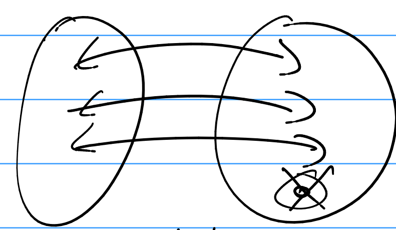
$$x + 0 = y - 5$$

add $\quad x + a \rightarrow$ a's add. inv is $-a$

mult $\quad x \cdot a \rightarrow$ a's mult. inv is $1/a$

Comp. $\quad f \circ g$ $\quad$ Need f





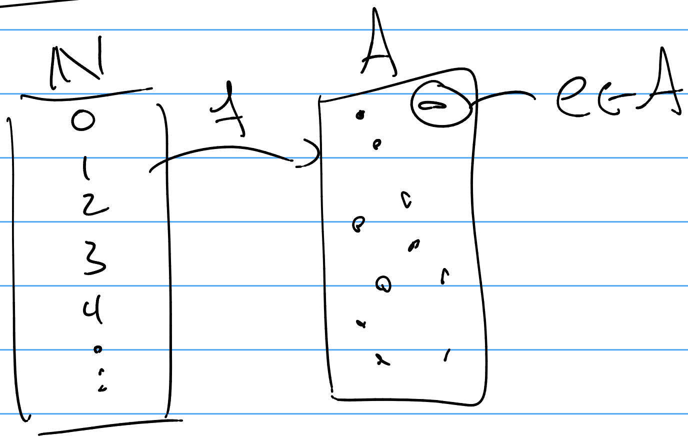
f^{-1}

For f^{-1} to exist f must be 1 to 1 and onto
(bijection)

in this class we will just check $(f \circ f^{-1})(x) = x$

$$(f^{-1} \circ f)(x) = x$$

Sequences / Series $\quad f : \mathbb{N} \to A$



ordered pairs

$0 \to e_0 = f(0)$
$1 \to e_1 = f(1)$
$2 \to e_2 = f(2)$
$3 \to e_3 = f(3)$
$\vdots$
$n \to e_n = f(n)$
$\vdots$

Make it "look" better,
write function as $e_0, e_1, e_2, e_3, \ldots$ (only write images in order)

sequence

ex **Notation:** $\{\text{rule}\}_{n=\square}$ 1^{st} number for index

index variable

$$\{3n+2\}_{n=0} = 2, 5, 8, 11, \ldots$$

Special Rules we should know

$$f(n) = a_n = c \cdot n + d$$

arithmetic seq

$$\{c \cdot n + d\}_{n=0} = d, \overset{+c}{d+c}, \overset{+c}{d+2c}, \ldots$$

$$\{a \cdot r^n\}_{n=0} = a \xrightarrow{\times r} ar \xrightarrow{\times r} ar^2, ar^3$$ geometric seq

$\{n\}_{n=1} = 1, 2, 3, \ldots$

$\{n!\}_{n=0} = 1, 1, 2, 6,$

$\{n^2\}_{n=1} = 1, 4, 9, \ldots$

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$\{n^3\}_{n=1} = 1, 8, 27, \ldots$

def $0! = 1$

Apps

① rule → seq

② seq → rule

Note: two types of rules to generate a seq.

① closed   ex $a_n = 3n^2 + n \qquad n = 0, 1, 2, \ldots$

$$a_2 = 3 \cdot 4 + 2 = 14$$

② open / recursive / inductive form

$$a_n = (\text{formula based on older known values})$$

$f_0 = 0 \quad f_1 = 1 \qquad f_n = f_{n-1} + f_{n-2}$

$$0, 1, 1, 2, 3, 5, 8, \ldots$$

add