Math 321

Q's

2.2 #9 show $A \cup \bar{A} = U$ $\qquad$ $A \cap \bar{A} = \emptyset$

#1 Set builder: $A \cup \bar{A} = \{e \mid e \in A \vee e \in \bar{A}\}$

$= \{e \mid \underbrace{e \in A}_{p} \vee \neg(\underbrace{e \in A}_{p})\}$ $\quad$ see $p \vee \neg p \equiv T$

$= \{e \mid T\}$

$= U$

#2 Subsets: show case 1 $\boxed{A \cup \bar{A} \subseteq U}$ $\xrightarrow{\text{logic}}$ $e \in A \cup \bar{A} \rightarrow \underbrace{\boxed{e \in U}}_{T}$

case 2 $U \subseteq A \cup \bar{A}$

#3 Membership tables

A	$\bar{A}$	$A \cup \bar{A}$
1	0	1
0	1	1

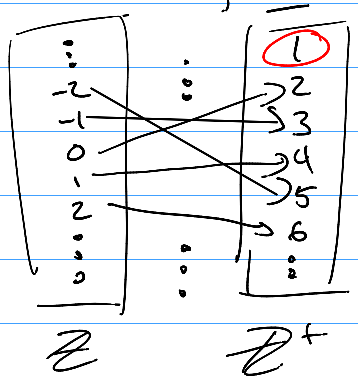
A	U
1	1
0	1

Same regions means $A \cup \bar{A} = U$

2.3 #21 Make a function that is ... $\qquad$ one-to-one (uniq. pairs)

$f: \mathbb{Z} \rightarrow \mathbb{Z}^+$ $\qquad$ rule

a) one-to-one, not onto $\qquad$ onto



rule in words

a) take 0, 1, 2, ... (non-neg ints) to evens.

b) take -1, -2, -3, ... (neg ints) to odds starting at 3.

codomain = range

explicit formula: $f(n) = \begin{cases} 2(n+1) & n \geq 0 \\ 2(n)+1 & n < 0 \end{cases}$

$f(0) = 2(0+1) = 2$
$f(1) = 2(1+1) = 4$

Seq's $f: \mathbb{N} \to S$ $\boxed{s_0, s_1, s_2, s_3, \ldots}$ Seq

formula: $\{f(n)\}_{n=0}$
$s_n = f(n)$

Formulas to know: ex $\{n^2\}_{n=0} = 0, 1, 4, 9, \ldots$

arithmetic $\{an+b\}_{n=0} = b, b+a, b+2a, \ldots$ ($+a$, $+a$)

geometric $\{ar^n\}_{n=0} = a, ar, ar^2, ar^3, \ldots$ ($\times r$, $\times r$, $\times r$)

Open vs closed formulas

recursion, iterative, inductive, recurrence relation

2 part process: ① Basis: give first elements of seq
② recurrence relation / Inductive Step
new values = expression of old values.

ex basis: $a_0 = 1$
inductive: $a_n = 2a_{n-1}$; $n = 1, 2, 3, \ldots, 30$

Seq: $1, 2, 4, 8, 16, \ldots$
$\uparrow$ 0 $\uparrow$ 1 $\uparrow$ 2 $\uparrow$ 3

closed form: $\{2^n\}_{n=0} = 2^0, 2^1, 2^2, \ldots$

Note: "Solve" a rec. relation. Means find a closed formula that generates the same seq.

how? $\boxed{\#1}$ guess, check

ex $a_n = 2a_{n-1}$ guess $a_n = 2^n \to f(n) = 2^n$

$$2^n \stackrel{?}{=} 2^1 \cdot 2^{n-1}$$

$$2^n \stackrel{?}{=} 2^n \quad \boxed{\text{true}}$$

Sums add up parts of a seq.

$$\{a_n\}_{n=0} = a_0, a_1, a_2, \ldots, \boxed{a_k, a_{k+1}, a_{k+2}, \ldots, a_\ell}, \ldots$$

Sum is just add parts of a seq

$$a_k + a_{k+1} + \ldots + a_\ell = \sum_{i=k}^{\ell} a_i$$

upper

lower

How to add (sum)?

① Add them

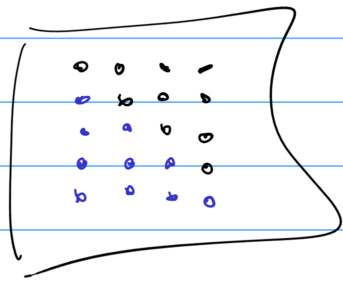
$$\sum_{k=3}^{21} \left(\frac{k^2+k}{7}\right) = \frac{3^2+3}{7} + \frac{4^2+4}{7} + \frac{5^2+5}{7} + \ldots + \frac{21^2+21}{7}$$

② Find a formula?

$$\sum_{k=1}^{100} k = 1+2+3+4+5+\dots+100$$

$$\begin{array}{l} 1+2+3+\dots+100 \\ 100+99+98+\dots+1 \\ \hline 101+101+101+\dots+101 = 100(101) \end{array}$$

$$= \frac{100(101)}{2} = 50(101) = \boxed{5050}$$



$$1+2+3+4 = \frac{4(5)}{2} = 2(5) = \boxed{10}$$

① $\sum_{k=1}^{n} k = 1+2+\dots+n = \frac{n(n+1)}{2}$

② $\sum_{k=1}^{n} k^2 = \frac{1}{3}\left[\frac{n(n+1)}{2}\right](2n+1) = \frac{n(n+1)(2n+1)}{6}$

③ $\sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2$