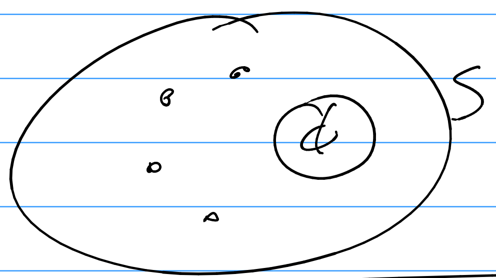
Math 321

Q's 2.1 #11 f $\boxed{\emptyset}\,\boxed{\in}\,\{x\}$ $\Rightarrow$ $\emptyset \subseteq \{x\}$

True

Meanings

$\emptyset = \{\}$ is the set of one element (the letter x)



$\emptyset \subseteq S$ $\quad$ $\emptyset \notin S$

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vs

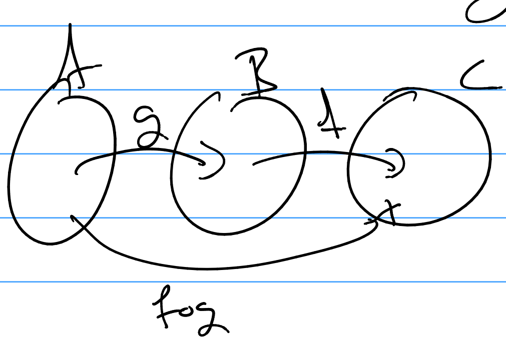
$\emptyset \in \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$ True

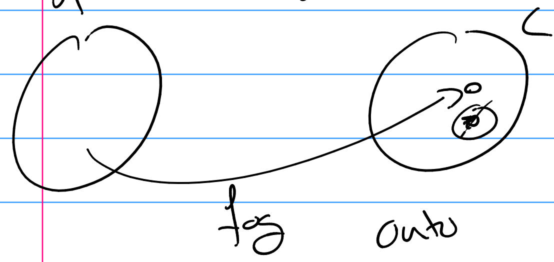
$\emptyset \subseteq$ ''

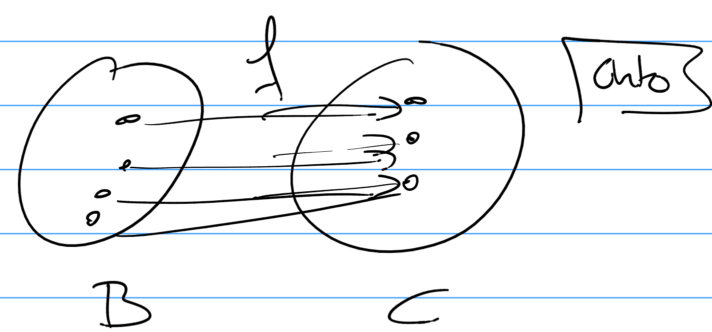
2.3 #35

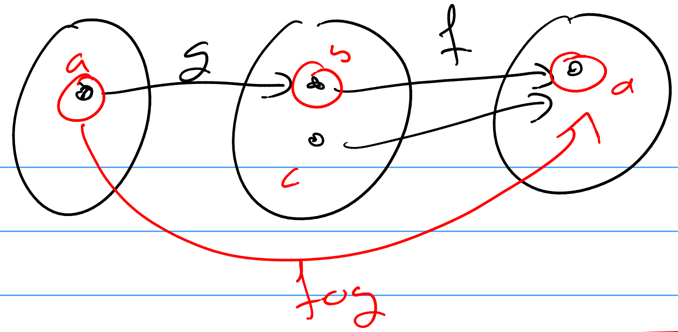
f and $\boxed{f \circ g}$ are onto

is g onto?









f is onto

g is not onto

fog is onto

no!

2.5 $|S|$ cardinality of a set

if $|S| = n \in \mathbb{N}$, call S finite

otherwise S is infinite

ex $\{1,2,3,\dots\}$ is infinite

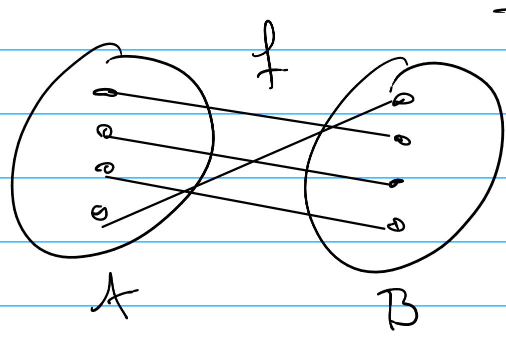
$$|\{1,2,3,\dots,101\}| = 101$$

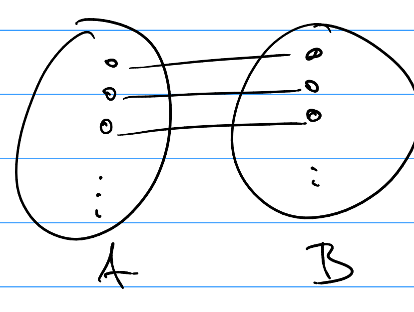
$$|\{a,b,c,\dots,z\}| = 26 \in \mathbb{N}$$

$|\mathbb{N}|$ is infinite ?

Compare Cardinalities $|A| = |B|$

Def $|A| = |B|$ if bijection from A to B exists





$f(n) = 3n$

$$\begin{aligned}
1 &\longrightarrow 3\\
2 &\longrightarrow 6\\
3 &\longrightarrow 9\\
4 &\longrightarrow 12\\
5 &\longrightarrow 15\\
&\vdots \quad \vdots
\end{aligned}$$

$\{1, 2, 3, 4, 5, \ldots\} = A$

$\{3, 6, 9, 12, \ldots\} = B$

$|A| = |B|$

take an obvious start set $|\mathbb{Z}^+| = \aleph_0$ aleph null

$\boxed{\text{Def}}$ any set that is finite or $|S| = \aleph_0$ we will call it countable.

Hilbert's Grand Hotel

rooms: 1, 2, 3, 4, 5, 6, ...

∞ lodge group 0

∞ lodge group 1

2

0 1 2 6

$a_{01}, a_{02}, a_{03}, a_{04}, \ldots$

3 a_{11}, a_{12} 5 $a_{13}, a_{14}, \ldots$

$a_{21}, a_{22}, a_{23}, a_{24}, \ldots$

4 $a_{31}, a_{32}, a_{33}, a_{34}, \ldots$

$\vdots$

$|\text{Hotel}| = \aleph_0$

$|\infty \text{ number of } \infty \text{ groups}| = \aleph_0$

all countable

What about $\mathbb{Q} = \{ \frac{a}{b} \mid b \neq 0 \wedge a, b \in \mathbb{Z} \wedge \text{no common factors}\}$

0 → change denom

change num

1 2 3 6 7 8 9

$$\begin{array}{llllll}
\pm\frac{1}{1} & \pm\frac{1}{2} & \pm\frac{1}{3} & \pm\frac{1}{4} & \pm\frac{1}{5} & \pm\frac{1}{6} \ \ldots\\
4,5\ \pm\frac{2}{1} & \pm\frac{2}{2} & \pm\frac{2}{3} & \pm\frac{2}{4} & \ldots & \\
10,11\ \pm\frac{3}{1} & \pm\frac{3}{2} & \pm\frac{3}{3} & \pm\frac{3}{4} & \ldots & \\
\pm\frac{4}{1} & \pm\frac{4}{2} & \pm\frac{4}{3} & \pm\frac{4}{4} & & \\
\vdots & & & \vdots & &
\end{array}$$

all $\frac{a}{b}$ such that $b \neq 0 \wedge a, b \in \mathbb{Z}$

Note: diagonals: 0 is a 1st diagonal

101/102 all $\frac{a}{b}$ have their n^{th} diagonal has all $a+b=n$

by counting along diagonals (skip all non rationals)

then our bijection..

$1 \to 0$

$2 \to 1$

$3 \to -1$

$4 \to 2$

$5 \to -2$

$6 \to 1/2$

$7 \to -1/2$

$\vdots$

gives $|\mathbb{Q}| = \aleph_0$

Next Class is $|\mathbb{R}|$