Math 321

Q's 2.4 17a $\quad a_0 = 2$

$a_n = 3a_{n-1} \rightarrow a_n = f(n)$?

Forward Iteration

$a_0 = 2$

$a_1 = 3a_0 = 3\cdot 2$

$a_2 = 3a_1 = 3(3\cdot 2) = 3^2\cdot 2$

$a_3 = 3a_2 = 3(3(3\cdot 2)) = 3^3\cdot 2$

$a_4 = 3^4\cdot 2$

$\vdots$

$a_n = 3^n\cdot 2 \qquad \boxed{a_n = 2\cdot 3^n}$

check does $\underline{a_n = 2\cdot 3^n}$ solve $\underline{a_0 = 2}$ $\quad \underline{a_n = 3\cdot a_{n-1}}$

$a_0 = 2\cdot 3^0 = 2\cdot 1 = 2$

$2\cdot 3^n = 3^1(2\cdot 3^{n-1})$

$2\cdot 3^n = 2\cdot 3^n$ ✓

Backward Iteration $\quad a_n = 3a_{n-1} = 3(a_{n-2})$

$a_n = 3a_{n-1}$

$a_n = 3(3a_{n-2}) = 3^2 a_{n-2}$

$a_n = 3(3(3a_{n-3})) = 3^3 a_{n-3}$

$a_n = 3^4 a_{n-4}$

$a_n = 3^5 a_{n-5}$

$a_n = 3^n \underset{n-n}{a_0} = \boxed{2\cdot 3^n}$

$$\sum_{k=1}^{n} 1 = \underset{k=1}{1} + \underset{k=2}{1} + \underset{k=3}{1} + 1 + \dots + \underset{k=n}{1} = n \cdot 1 = n$$

$$\sum_{k=1}^{n} k = ?$$

$\boxed{\text{telescoping sum}}$

① ex $\displaystyle\sum_{k=1}^{4}\left(\frac{1}{k} - \frac{1}{k+1}\right) = \left(\frac{1}{1} - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{4}} - \frac{1}{5}\right)$

$= \boxed{1 - \frac{1}{5}}$

② $(k+1)^2 - (k)^2 = k^2 + 2k + 1 - k^2 = 2\boxed{k} + 1$

#38 use $\displaystyle\sum_{k=1}^{n} 1 = n \quad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

use $(k+1)^3 - k^3 = k^3 + 3k^2 + 3k + 1 - k^3$

$(k+1)^3 - k^3 = 3k^2 + 3k + 1$

So $\displaystyle\sum_{k=1}^{n} (k+1)^3 - k^3 = \sum_{k=1}^{n} 3k^2 + 3k + 1$

$\displaystyle\boxed{\sum_{k=1}^{n} (k+1)^3 - k^3} = 3\boxed{\sum_{k=1}^{n} k^2} + 3\boxed{\sum_{k=1}^{n} k} + \boxed{\sum_{k=1}^{n} 1}$

$\| \qquad\qquad \| \qquad\qquad \|$

$(\cancel{2^3} - 1^3) + (\cancel{3^3} - \cancel{2^3}) + (\cancel{4^3} - \cancel{3^3}) + \dots + (n+1)^3 - \cancel{k^3} \qquad \frac{n(n+1)}{2} \qquad n$

$\|$

$(n+1)^3 - 1$

Exam 11 probs @ 10pts, 100pts = 100%

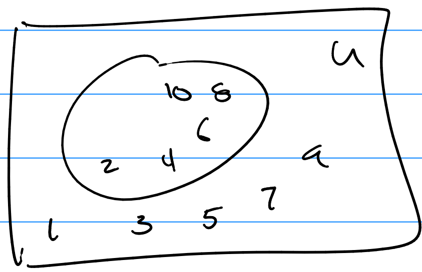
2.1 Intro (2 probs)

① given sets and U.D → write sets in roster (list), set-builder, and Venn

$U = \{1, 2, \ldots, 10\}$

A is evens — roster $A = \{2, 4, 6, 8, 10\}$

$= \{e \mid e = 2k \wedge k \in \mathbb{Z} \wedge 1 \leq k \leq 5\}$



② 20 cross product

20 power set

2.2 Set ops (2 probs)

① Membership Table (+) Venn

② Equality of Sets using set builder notation (+) logic

2.3 Functions (2 probs)

① p. 154 33, 34 or 35

② find functions that are ... one to one and onto?

$f: A \to B$

$\neg$ one to one and onto?

one to one and $\neg$ onto?

$\neg$ one to one and $\neg$ onto?

2.4 Seq, Sum (3 probs)

① a) formula → seq

b) seq → formula

② Sum by formula know: $\sum_{k=1}^{n} 1$, $\sum_{k=1}^{n} k$, $\sum_{k=1}^{n} k^2$, $\sum_{k=0}^{n} ar^k$

ex $\sum_{k=21}^{999} (k+1) = \sum_{k=1}^{999} (k+1) - \sum_{k=1}^{20} (k+1)$

$\sum_{k=1}^{999} k + \sum_{k=1}^{999} 1 - \sum_{k=1}^{20} k - \sum_{k=1}^{20} 1$

$22 + 23 + 24 + \ldots + 1000$

$1000 + 999 + 998 + \ldots + 22$

$1022 + 1022 + 1022 + \ldots + 1022 = (1022)(1000 - 22 + 1)$

③ telescoping sum

2.5 Cardinality (2 probs)

① Prove $\mathbb{Q}$ is countable

② Prove $\mathbb{R}$ is uncountable

2.6 Not on exam (but ... know it for next semester)

(Matrices) starts from solve $\begin{cases} 2x + y = 3 \\ x - 4y = 7 \end{cases}$ linear system of eqn's

$$\begin{bmatrix} 2 & 1 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

For us two matrix types ① all values are real numbers

② all values are bits, 0 or 1.

(zero-one matrix)

ops $A+B$, $A*B$, A^n

① Add
$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 2 & -3 \end{bmatrix} = \boxed{\begin{bmatrix} 1 & 1 \\ 5 & 1 \end{bmatrix}}$$

② mult
$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 0 & 4 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 6 \\ 0 & 16 \end{bmatrix}$$

③ power
$$A^n = \underbrace{A \cdot A \cdot A \cdot A \cdot \ldots \cdot A}_{n\text{-times}}$$

Zero-one matrices (bits 0, 1)

① join $A \vee B$ (bitwise or)
$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \vee \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

② meet $A \wedge B$ (bitwise and)
$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \wedge \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

③ boolean product
$$A \odot B$$