# Math 321

2.4 #13a $a_n = -3a_{n-1} + 4a_{n-2}$

$a_n = 0$ [check] plug in the function and see if $=$ is true.

$0 \stackrel{?}{=} -3(0) + 4(0)$

$0 \stackrel{?}{=} 0$ [true]

---

toys: $\mathbb{Z}$ (classically $\mathbb{Z}^+$)

$\{\ldots, -2, -1, 0, 1, 2, \ldots\}$ $\{1, 2, 3, 4, \ldots\}$

rules: Same? → two types: ① equality ② congruence

operations: $+$, $*$

division (arithmetic) $\frac{1}{3} = 0.333\ldots$

divides (about "sharing") is a propositional function

a divides b

$a \cdot c = b$

$c \in \mathbb{Z}$

4 divides 8 is true

why? $4 \cdot 2 = 8$

ex 3 divides 9 [true] b/c $3 \cdot 3 = 9$

4 divides $-12$ [true] b/c $4(-3) = -12$

symbols: $3 \mid 9$ $3 \nmid 10$ is $\neg(3 \mid 10)$

$6 \nmid 3$ $7 \nmid 3$

Properties of $a|b$ (remember this means $a \cdot c = b$, $c \in \mathbb{Z}$)

**Thm** $a, b, c \in \mathbb{Z}$, $a \neq 0$

① $a|b \wedge a|c \rightarrow a|(b+c)$

**pf** $a \cdot k_1 = b \wedge a \cdot k_2 = c \rightarrow (b+c) = a \cdot k_1 + a \cdot k_2 = a(k_1+k_2)$

$a|b+c$

$a \cdot (j+t) = (b+c)$

② $a|b \rightarrow a|b \cdot c$

③ $a|b \wedge b|c \rightarrow a|c$

---

What if $a|b$ or $a \nmid b$? Can we put this into the concept?

~~16~~
~~13~~
10

| $p_1$ | $p_2$ | $p_3$ | |
|---|---|---|---|
| 1 | 1 | 1 | $=3$ |
| 1 | 1 | 1 | $=3$ |

$\geq 3$

Division Algorithm

$16 = 5 \cdot 3 + (1)$

$7 = 1 \cdot 5 + (2)$

$16 = 1 \cdot 3 + 13$

$5 \nmid 7$

$\frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}$

$\frac{1}{3}\ \frac{1}{3}\ (\frac{1}{3})$

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ |
| $\frac{1}{15}$ | $\frac{1}{15}$ | $\frac{1}{15}$ | $\frac{1}{15}$ | $\frac{1}{15}$ |

$\frac{7}{5} = 1 + \frac{1}{3} + \frac{1}{15}$

$b = q \cdot a + r$ (dividend, quotient, divisor, remainder)

$0 \le r < a$

$a \mid b$ means $r = 0$

$a \nmid b$ means $r \neq 0$

**ex** try 4 divides 22 $\quad 22 = 5 \cdot 4 + 2$

4 divides $-22$ $\quad -22 = (-6)4 + 2$ (quotient, remainder)

Modulus: $b \bmod a = r$ if $b = qa + r$

div: $b \operatorname{div} a = q$ if $b = qa + r$

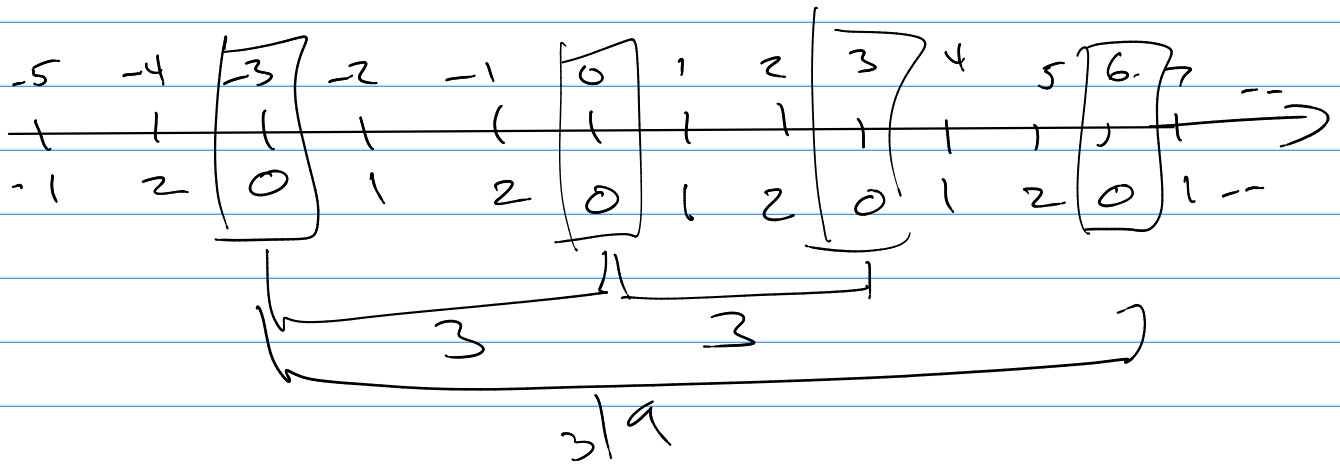
$22 \bmod 3 = 1$ $\quad -22 \bmod 3 = 2$

$22 \operatorname{div} 3 = 7$ $\quad -22 \operatorname{div} 3 = -8$

$22 = 7 \cdot 3 + 1$ $\quad -22 = -8 \cdot 3 + 2$

---

New Same: Congruence




**Def** a is congruent to b under modulo m

if $m \mid (a - b)$