

Math 321

Q5

4.1 #3

$$a|b \rightarrow a|bc, c \in \mathbb{Z}$$

means

$$a \cdot k_1 = b, k_1 \in \mathbb{Z}$$

$$\rightarrow a \cdot k_2 = bc, k_2 \in \mathbb{Z} \text{ goal}$$

PF

Direct

assume $a|b$. by def is $a \cdot k_1 = b, k_1 \in \mathbb{Z}$

$$\rightarrow a \cdot k_1 \cdot c = b \cdot c \rightarrow a|bc$$

integer by closure prop. of mult. & ints

Understand: " q is a mult. of s "

in algebra: $s \cdot c = q, c \in \mathbb{Z}$

in Number theory: $s|q$

4.2 #27

$$3^{2003} \pmod{99}$$

2003 products

$$243 \pmod{99} = 45$$

slow tech:

$$(3 \cdot 3 \cdot 3 \cdot 3 \cdot 3) \cdot 3 \cdot \dots \cdot 3 \pmod{99}$$

$9 \sqrt{27} \cdot 3 \cdot \dots \cdot 3$
 $27 \cdot 3 \cdot \dots \cdot 3$
 $81 \cdot 3 \cdot \dots \cdot 3$
 $243 \cdot 3 \cdot \dots \cdot 3$
 $45 \cdot 3 \cdot \dots \cdot 3$

fast:

$$2003 = (?)_2$$

	div	mod
2003	1001	1
1001	500	1
500	250	0
250	125	0
125	62	1

$$2003 = (1, 1, 1, 1, 1, 0, 1, 0, 0, 1)_2$$

62	31	0	↑	3	2003	mod 99
31	15	1				
15	7	1				
7	3	1				
3	1	1				
1	0	1				

$() () () () () () (3^{32}) (3^{16}) (3^8) (3^4) (3^2) (3)$ mod 99
 81 81 9 3

	base	ans
1	3	
1	9	
0	81	
0	$81^2 \equiv 27$	
1	$27^2 \equiv 81$	
0		
1		

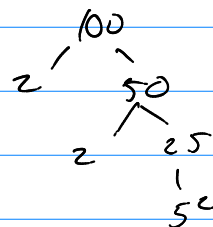
Fund. th^m for all $n \geq 2$, n is prime or a uniq prod. of primes written in non-dec. order

Fund. th^m
 $1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, \dots$
 $\rightarrow (2), (3), (2^2), (5), (2 \cdot 3), (7), (2^3), \dots$

Findy Prime Factors

Th^m if n is composite, then n has a prime factor, p , such that $2 \leq p \leq \sqrt{n}$

(ex) **Factor 100** \rightarrow check: 2, 3, 5, 7
 $100 = (2^2 5^2)$



factor 101

check: 2, 3, 5, 7

2x101 3x101 5x101 7x101
+ 101 is prime

Prime Sieves

1, 2, 3, 4, 5, 6, 7, 8, 9, 10
11, 12, 13, 14, 15, 16, 17, 18, 19, 20
21, 22, 23, 24, 25, 26, 27, 28, 29, 30,
31, ..

Thⁿ infinitely many primes.

PF (by contradiction)

assume finite primes: $p_1, p_2, p_3, \dots, p_n$ n is finite.

consider: $P = p_1 \cdot p_2 \cdot p_3 \cdot \dots \cdot p_n + 1$

by fact, there's some prime (call it p_*) must divide P ,
so $p_* \mid P$ and $p_* \mid (p_1 \cdot p_2 \cdot \dots \cdot p_n)$

so $p_* \mid P - (p_1 \cdot p_2 \cdot \dots \cdot p_n) \rightarrow p_* \mid 1 \equiv \text{F}$

Density

Def: $\pi(n) = (\text{counts number of primes } \leq n)$

ex $\pi(10) = 4$, $\pi(11) = 5$, $\pi(12) = 5, \dots$

Thⁿ

$\lim_{n \rightarrow \infty} \frac{\pi(n)}{n/\ln(n)} = 1$ so for large n $\pi(n) \sim \frac{n}{\ln(n)}$

% of primes $\frac{\pi(n)}{n} \sim \frac{n/\ln(n)}{n} = \frac{1}{\ln(n)}$