Math 321

Q's

Note: Linked video playlist on Black board for Induction has 5 examples in it.

Strong Induction

Basis Step: prove the 1^{st} element (1^{st} cases)

Inductive Step: prove...

$\big(P(1^{st} \text{case}) \wedge P(2^{nd} \text{case}) \wedge \dots \wedge P(k^{th} \text{case})\big) \to P(k+1^{st} \text{case})$

Ex

$\boxed{\text{Fund th}^{m} \text{ of Arith}}$ for $n \geq 2$, n is prime or a uniq prod of primes in non-dec. order.

to prove this we need two things

1. existence: $n \geq 2$, n is prime or a prod. of primes
2. uniqueness

$\boxed{\text{prove existence}}$ $n \geq 2$ is prime or a prod. of primes (use strong induction)

pf **Basis Step:** prove $P(1^{st}$ case$)$

$n = 2$ is a prime, so True

Inductive Step: $(P(1^{st}) \wedge \dots \wedge P(k)) \to P(k+1)$

If $2, 3, 4, 5, \dots, k$ are prime or prod. of primes

I.H.

then $k+1$ is prime or prod. of primes

Scratch Paper

$P(1^{st}$ case$)$: $n = 2$ is prime so True

$P(2^{nd}$ case$)$: $n = 3$ is prime so True

$P(3^{rd}$ case$)$: $n = 4$ 2^2 so True

$\vdots$

$P(k^{th}$ case$)$: $n = k$

$P(k+1^{st}$ case$)$: $n = k+1$

assume $\boxed{2, 3, 4, \ldots, k \text{ are prime or prod. of primes.}}$ I.H.

Now: what about $k+1$? It's a number so it's prime or composite

case 1 $k+1$ is prime. True statement

case 2 $k+1$ is composite. By def. some numbers a, b

$2 \le a \le k$, $2 \le b \le k$ are $a \cdot b = k+1$

by I.H., a is prime or prod. of primes, b is prime or prod. of primes

so $a \cdot b$ is a prod. of primes $\therefore$ True.

Uniqueness proof (contradiction proof)

assume n = some prod of primes $= p_1 p_2 p_3 \cdots p_N$

n = some other prod. of primes $= q_1 q_2 q_3 \cdots q_M$

Know: ① primes p_i, p_j $p_i \neq p_j$ $p_i \mid p_j \equiv F$

② prime | prod of primes says one of the primes in the product must be the dividing prime.

(see video)

Induction:

Basis Step: 1^{st} case(s)

Inductive Step: $P(1^{st}) \wedge P(2^{nd}) \wedge \ldots \wedge P(k^{th}) \rightarrow P(k+1^{st})$

proves: $P(1) \wedge P(2) \wedge P(3) \wedge \ldots$

we can use this to make sets (Recursive Definitions)

ex **Seq**

$f_0 = 0$, $f_1 = 1$ ← Basis Step

$f_n = f_{n-1} + f_{n-2}$; $n = 2, 3, 4, \ldots$ ← Inductive Step

Fibonacci Numbers

$0, 1, 1, 2, 3, 5, 8, \ldots$

Before: Open or Closed representations of seq's

ex $\quad 1, 2, 4, 8, 16, \ldots$

① closed: $\{2^n\}$ $\quad n = 0, 1, 2, \ldots$

② open (recursive def)

$a_0 = 1 \longleftarrow$ Basis

$a_n = 2 \cdot a_{n-1} \quad n = 1, 2, 3, \ldots \longleftarrow$ Induction Step

$a_n = f(a_{n-1}, a_{n-2}, \ldots, a_0)$

$f_n = f_{n-1} + f_{n-2} \qquad f_0 = 0, \; f_1 = 1$

$$f(n) = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$$

$0, 1, 1, 2, 3, 5, \ldots$

Know: $f_n = f_{n-1} + {}_{n-2} \quad ; \quad f_0 = 0, \; f_1 = 1$

show $f_1^2 + f_2^2 + f_3^2 + \cdots + f_n^2 = f_n f_{n+1}$

1st case $\quad f_1^2 = f_1 f_2$

2nd case $\quad f_1^2 + f_2^2 = f_2 f_3$

3rd case $\quad f_1^2 + f_2^2 + f_3^2 = f_3 f_4$

$\vdots$

k^{th} $\quad \vdots$

$k+1^{st}$ $\quad \vdots$