Math 321

Extra Credit (Due Monday)

Q's — 5.1 #10 $\quad \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \cdots \frac{1}{n(n+1)} = \frac{n}{n+1}$

Case 1 $\quad \frac{1}{1\cdot 2} = \frac{1}{2}$

Case 2 $\quad \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} = \frac{2}{3}$

Case 3 $\quad \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} = \frac{3}{4}$

Case 4 $\quad \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5} = \frac{4}{5}$

$\vdots$

Case K $\quad \frac{1}{1\cdot 2} + \cdots + \frac{1}{K(K+1)} = \frac{K}{K+1}$

Case K+1 $\quad \frac{1}{1\cdot 2} + \cdots + \frac{1}{K(K+1)} + \frac{1}{(K+1)(K+2)} = \frac{K+1}{K+2}$

If

Basis Step — prove 1st case $\quad \frac{1}{1\cdot 2} \overset{?}{=} \frac{1}{2}$ is true.

Inductive Step — assume K^{th} case $\quad \frac{1}{1\cdot 2} + \cdots + \frac{1}{K(K+1)} = \frac{K}{K+1}$ I.H.

(goal: show $\frac{1}{1\cdot 2} + \cdots + \frac{1}{K(K+1)} + \frac{1}{(K+1)(K+2)} = \frac{K+1}{K+2}$)

Start with $\left(\frac{1}{1\cdot 2} + \cdots + \frac{1}{K(K+1)}\right) + \frac{1}{(K+1)(K+2)}$

$\overset{\text{I.H.}}{=} \left(\frac{K}{K+1}\right) + \frac{1}{(K+1)(K+2)} = \frac{K(K+2)+1}{(K+1)(K+2)} = \frac{K^2+2K+1}{(K+1)(K+2)}$

$= \frac{(K+1)(K+1)}{(K+1)(K+2)} = \frac{K+1}{K+2}$ true

Counting chapter 6

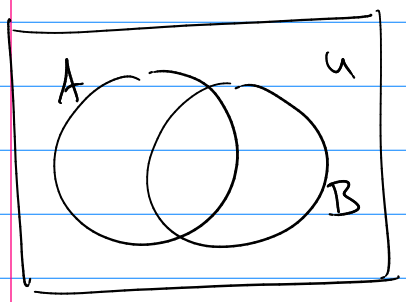
Q $|S| = ?$

"small" problem you $|\{e_1, e_2, e_3, \ldots, e_n\}| = n$

- count the elements.
- count process that creates all the elements.
  - i) don't miss elements
  - ii) don't over count
    - ex - sit 4 people at table
      - decide same people = same sitting

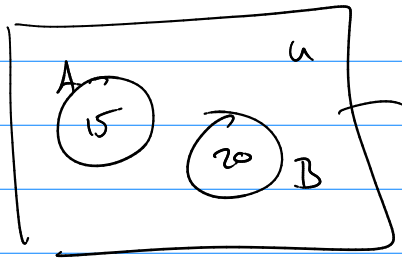
6.1 Basics

① Sum Rule count elements from A or B.



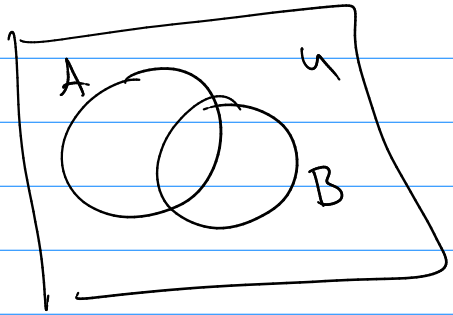
$|A \cup B| = |A| + |B|$

if



$A \cap B = \emptyset$ (disjoint)

② what if $A \cap B \neq \emptyset$ (not disjoint)



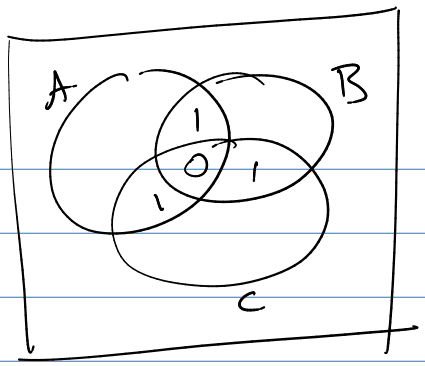
ex $|A| = 15$ $|B| = 20$

but I know there are $|A \cap B| = 5$

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Subtraction Rule or Inclusion / Exclusion Principle

Q1



if disjoint $|A \cup B \cup C| = |A| + |B| + |C|$

not disjoint (Inclusion / Exclusion)

A	B	C
1	1	1
1	1	0
1	0	1
1	0	0
0	1	1
0	1	0
0	0	1
0	0	0

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

Q2 You ask 3 questions (yes/no)

breakfast	$\|Q_1\| = 15$	$\|Q_1 \cap Q_2\| = 10$	$\|Q_1 \cap Q_2 \cap Q_3\| = 1$
coffee	$\|Q_2\| = 20$	$\|Q_1 \cap Q_3\| = 3$	
exercise	$\|Q_3\| = 5$	$\|Q_2 \cap Q_3\| = 2$	

total people $= |Q_1| + |Q_2| + |Q_3| - |Q_1 \cap Q_2| - |Q_1 \cap Q_3| - |Q_2 \cap Q_3| + |Q_1 \cap Q_2 \cap Q_3|$

$= 15 + 20 + 5 - 10 - 3 - 2 + 1 = 26$