Math 321

Basics of Counting

① Sum Rule (Key word: "or")

task: pick A or B $\rightsquigarrow$ $|A \cup B| = |A| + |B|$ if disjoint

② (non-disjoint) $|A \cup B| = |A| + |B| - |A \cap B|$

(Subtraction Rule)

↳ Inclusion Exclusion Principle ...

$$|A_1 \cup A_2 \cup \dots \cup A_n| = |A_1| + |A_2| + \dots + |A_n| - |A_1 \cap A_2| - \dots - |A_{n-1} \cap A_n| + |A_1 \cap A_2 \cap A_3| + \dots - \dots + \dots (+/-) |A_1 \cap A_2 \cap \dots \cap A_n|$$

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③ Product Rule

task: pick from A and then from B. (you have a pair of objects)

2 sets: $|\text{all possible pairs}| = |A \times B| = |A| \cdot |B|$

n-sets: $|A_1 \times A_2 \times \dots \times A_n| = |A_1||A_2| \dots |A_n|$

ex) How many passwords of 10 symbols if 1st is a capital, 2nd is a digit, and last 3 a lower case.

ex) $10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$? . 5^3

pick 7 symbols

(pick a capital) and (pick a digit) and [ ] and (pick lowercase)

$26 \cdot 10 \cdot \text{Symbol}^7 \cdot 26 = 95^7 \cdot 26^2 \cdot 10$

Symbol: lower or upper or digit or other

$26 + 26 + 10 + 33 = 95$

(a) **Division Rule**

|task| = n ways to do task

but for every way, w, exactly d of the n ways correspond to w.

then $\frac{n}{d}$ ways to do the task.

(ex) 6 people sitting in a row. How many ways?

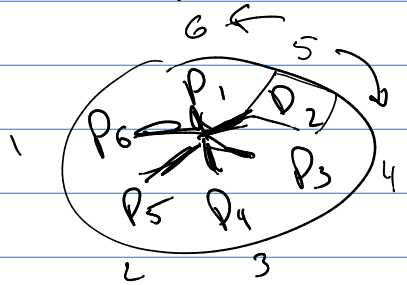
$$\frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1} = 6!$$

$P_1\ P_2\ P_3\ P_4\ P_5\ P_6$

Notation: $n! = n \cdot (n-1)(n-2) \cdots 1$

but $0! = 1$

(vs) 6 people in a circle but sitting clockwise or counterclockwise is the same.



$6!$ ways

$$\frac{6!}{6 \cdot 2} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 2} = 60$$

Ex 4 lowercase letters

a) how many if you can repeat letters? $26 \cdot 26 \cdot 26 \cdot 26 = \boxed{26^4}$

b) no repeats? $\boxed{26 \cdot 25 \cdot 24 \cdot 23}$

or $\frac{26!}{22!} = \frac{26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \cdots 1}{22 \cdot 21 \cdots 1}$

c) start and end with 'b' and letters can be repeated.

$1 \cdot 26 \cdot 26 \cdot 1 = \boxed{26^2}$

d) contains abc (in that order)

front or	abc · something	$1 \cdot 26$
		+
end	something · abc	$26 \cdot 1$
		$\boxed{52}$

e) _ a _ b _ c _ $\quad 1 \cdot 4 \cdot 26$

Applications of Counting

① percentage of probability of an event $= \frac{|\text{Event}|}{|\text{all possible things}|}$

② 6.2 Pigeonhole Principle

Thm $k \in \mathbb{Z}^+$. If $k+1$ or more objects are placed into k boxes, then at least one box as at least 2 objects.

generalized

Pigeonhole Principle $k \in \mathbb{Z}^+$. If N objects are placed into k boxes, then at least one box has at least $\lceil \frac{N}{k} \rceil$ objects