Math 321

6.2 Pigeonhole Principle

Thm $k \in \mathbb{Z}^+$. If $k+1$, or more, objects are placed into k boxes, then at least one box has at least 2 objects.

(generalized)

Thm $k \in \mathbb{Z}^+$. If N objects are placed into k boxes, then at least one box has at least $\lceil \frac{N}{k} \rceil$ objects.

ex How many students do I need to know at least 3 have same first, last initials (ex me → M.a)

all initials a.a, a.b, ..., z.z is 26^2

to do these problems we need to understand **and** how many.

object = ??
boxes = ??

$|\text{boxes}| = 26^2$ (possible initials)

objects = students So $|\text{students}| = N$ ← find.

when $\left\lceil \frac{N}{26^2} \right\rceil = 3$

$N = 2 \cdot 26^2 + 1$

$\lceil \frac{1}{3} \rceil = 1 \quad \lceil \frac{2}{3} \rceil = 1 \quad \lceil \frac{3}{3} \rceil = 1$

$\lceil \frac{4}{3} \rceil = 2 \quad \lceil \frac{5}{3} \rceil = 2 \quad \lceil \frac{6}{3} \rceil = 2$

$\lceil \frac{7}{3} \rceil = 3 \quad \lceil \frac{8}{3} \rceil = 3 \quad \lceil \frac{9}{3} \rceil = 3$

$\lceil \frac{10}{3} \rceil = 4$

(ex) how many socks (picked blind) do you need if you have red, black, grey, and white socks to get 3 pairs of the same color?

6 socks of same color

objects = socks
boxes = colors

$\lceil \frac{N}{4} \rceil = 6$

$N = 5 \cdot 4 + 1 = \boxed{21}$

(ex) 11 black socks, 9 red, 13 white, 5 grey

→ how many (picked blind) to get 1 pair of black socks.

(not pigeonhole principle problem it is a worst case problem)

= pick red then white then grey and finally black

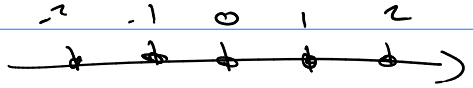
= $9 + 13 + 5 + 2$ (finally get black pair)

= $\boxed{29}$

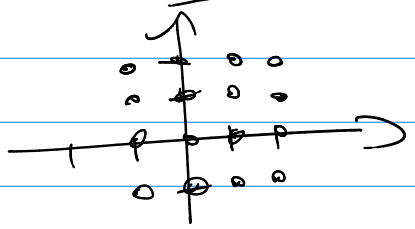
(ex) $\mathbb{Z}$, $\mathbb{Z} \times \mathbb{Z}$, $\mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$ ... } Space Dimensions

$\mathbb{Z}^1$, $\mathbb{Z}^2$, $\mathbb{Z}^3$, ...

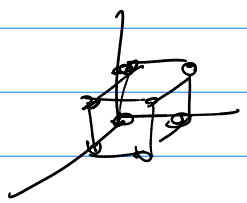
$\mathbb{Z}$



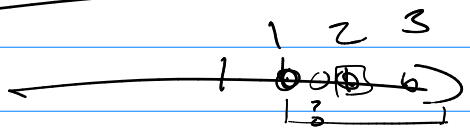
$\mathbb{Z}^2$



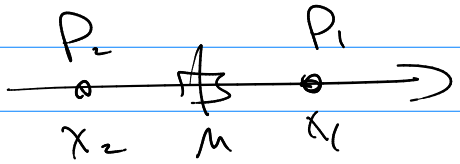
$\mathbb{Z}^3$



where is a midpoint?



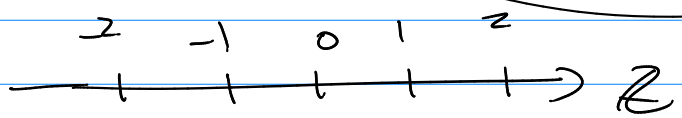
(1D)



$M = \frac{x_1 + x_2}{2}$

How many points do you need to pick, to know one of the midpoints would also have integer coordinates.

(1D) $p_1 = x_1$, $p_2 = x_2$



$M = \left(\frac{x_1+x_2}{2}\right)$ ← means $\frac{x_1+x_2}{2}$ is an int.

So integer midpoint says $x_1 + x_2 \equiv$ even

know

x_1	x_2	
even	even	= even
odd	odd	= even
odd	even	= odd
even	odd	= odd

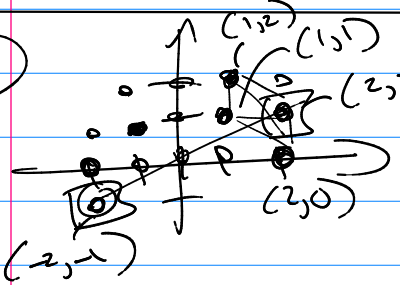
← mean int. midpt.

boxes = even or odd

$|\text{boxes}| = 2$ if we have 3 or more points → some midpt has integer coord.

(why? b/c two will have same parity)

(2D)



$p_1 = (x_1, y_1)$ $p_2 = (x_2, y_2)$

$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

how many points so some midpoint has integer coord?

$|\text{boxes}| = 4$ — points are (even, even)
(even, odd)
(odd, even)
(odd, odd)

$|\text{objects}| =$ points → take 5 points to get two of same parity.