Math 321

Q's 6.1 #25 $d_1d_2d_3$ (decimal digits $= \{0,1,2,\ldots,9\}$)

a) do not contain the same digit 3 times.

Note: ① read, re-read, read again → what is the problem
② Sum rule & disjoint sets.

ex) $|A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3|$

↳ b/c this is an equality --

$$|A_1 \cup A_2 \cup A_3| - |A_1| - |A_2| = |A_3|$$

↳ $d_1d_2d_3$ and do not contain same digit 3 times.

$$|\text{all 3 digit } d_1d_2d_3| = |\{\text{do not contain same digit 3 times}\}| + |\{000, 111, 222, \ldots, 999\}|$$

$$10^3 = \boxed{\quad} + 10$$

$$\text{ans} = 10^3 - 10$$

b) $d_1d_2d_3$ and d_1 is an odd digit

$$5 \cdot 10 \cdot 10 = 5 \cdot 10^2$$

c) have exactly 2 digits that are 4's (not 1, not 3)

ex 440
414
044

exactly 2 4's

Process: get 4 4 and pick another digit and place the digit

$(1) \cdot (9) \cdot (3) = 9 \cdot 3 = 27$

(not 10 b/c 4 can't be used)

Applications:

1. Pigeonhole Principle
2. Apply Sum Rule in an interesting way (see above)

$|A \cup B| - |A| = |B|$ (if disjoint)

3. Apply the division rule $0 \le r \le n$

→ Permutations: select r objects from n objects and order matters

Combinations: select r obj. from n, and order does not matter.

Permutations: arrange n objects $= n!$

take r, so $(n-r)$ not selected so $(n-r)!$ is overcount → $\frac{n!}{(n-r)!}$

order matters

(ex) arrange 15 people $15!$

but 5 matter and 10 do not. $10!$ ways to arrange the 10 who do not affect task.

$$|\text{& 15 take 5 in order}| = \frac{15!}{10!}$$

Because we select r from n $(0 \le r \le n)$ in lots of problems... make it a function:

$$\boxed{P(n,r) = \frac{n!}{(n-r)!} = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot (n-r+1)}$$

(ex) $P(15,5) = \frac{15!}{10!} = \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot (10!)}{(10!)} = \boxed{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11}$

Combinations: arrange n objects : $n!$

but choose (without order) r objects → $r!$

so $(n-r)$ have not been chosen → $(n-r)!$

$$\frac{n!}{r!(n-r)!}$$

example

15 choose 5 without order

$P_1\ P_2\ P_3\ P_4\ P_5 \mid P_6\ P_7 \cdots P_{15}$

$5!$ $\quad$ $10!$

$$\frac{15!}{5!\ 10!}$$

b/c we do probs like this a lot ... make it a function

$$C(n,r) = \frac{n!}{r!(n-r)!}$$

Notation: $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

eg's 13 people. 9 guys, 4 girls

a) how many ways to pick (order matters) 5 to play basketball

$$P(13,5) = \frac{13!}{8!}$$

b) choose 5. (don't care about order)

$$\binom{13}{5} = \frac{13!}{5!\ 8!}$$

c) choose 5 but at least 1 must be a girl

all teams = no girls + exactly 1 girl + exactly 2 girls + exactly 3 girls + exactly 4 girls

$$\binom{13}{5} = \binom{4}{0}\cdot\binom{9}{5} + \binom{4}{1}\cdot\binom{9}{4} + \binom{4}{2}\cdot\binom{9}{3} + \binom{4}{3}\binom{9}{2} + \binom{4}{4}\binom{9}{1}$$

$$P(\text{at least 1 girl}) = \frac{\binom{13}{6}}{\binom{5}{0}\binom{8}{6}} - \frac{\binom{9}{6}}{\binom{5}{0}\binom{4}{6}}$$

$$= \frac{\binom{4}{6}}{\binom{1}{0}\binom{3}{0}} \cdot \frac{\binom{9}{6}}{\binom{4}{0}\binom{5}{0}} + \frac{\binom{4}{0}}{\binom{2}{0}\binom{2}{0}} \frac{\binom{9}{0}}{\binom{3}{0}\binom{6}{0}} + \frac{\binom{4}{6}}{\binom{3}{0}\binom{1}{0}} \frac{\binom{9}{0}}{\binom{2}{0}\binom{7}{0}} + \frac{\binom{4}{6}}{\binom{4}{0}\binom{0}{0}} \frac{\binom{9}{6}}{\binom{1}{0}\binom{8}{6}}$$