

Math 321

Q's

6.1 #25 $d_1 d_2 d_3$ (decimal digits = $\{0, 1, 2, \dots, 9\}$)

a) do not contain the same digit 3 times.

Note: ① read, re-read, read again \rightarrow what is the problem

② Same idea of disjoint sets.

ex) $|A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3|$

\hookrightarrow b/c this is an equality --

$$|A_1 \cup A_2 \cup A_3| - |A_1| - |A_2| = |A_3|$$

b) $d_1 d_2 d_3$ and do not contain same digit 3 times.

$$|\text{all 3 digit } d_1 d_2 d_3| = |\text{do not contain same digit 3 times}| + |\begin{matrix} 000 \\ 111 \\ 222 \\ \vdots \\ 999 \end{matrix}|$$

$$10^3 = \boxed{} + 10$$

ans = $10^3 - 10$

b) $d_1 d_2 d_3$ and d_1 is an odd digit

$$5 \cdot 10 \cdot 10 = \boxed{5 \cdot 10^2}$$

c) have exactly 2 digits that are 4's
not 1 not 3
 exactly 2-4's

ex 440
414
044

Process: get 44 and pick another digit and place the digit

$$(1) \cdot (9) \cdot (3) = 9 \cdot 3 = 27$$

not 10 b/c 4 can't be used

Applications:

- ① Pigeonhole Principle
- ② Apply Sum Rule in an interesting way (see above)

$$|A \cup B| - |A| = |B| \quad (\text{if disjoint})$$
- ③ Apply the division rule $0 \leq r \leq n$
 - Permutations: select r objects from n objects and order matters
 - Combinations: select r obj. from n , and order does not matter.

Permutation: arrange n objects = $n!$

take r , so $(n-r)$ not selected so $(n-r)!$ is overcount $\frac{n!}{(n-r)!}$

order matters

(ex) arrange 15 people $15!$

but 5 matter and 10 do not. $10!$ ways to arrange the 10 who do not affect task.

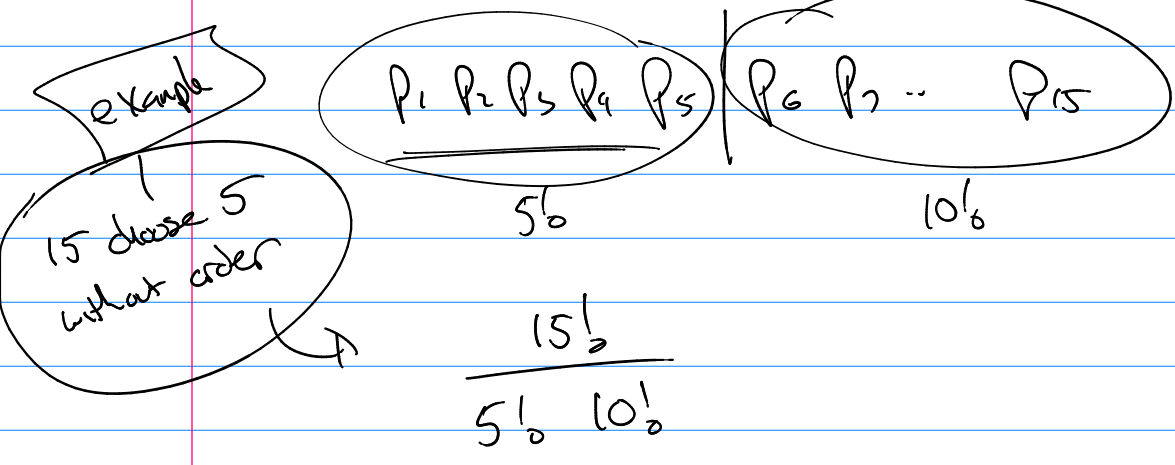
$$| \& 15 \text{ take 5 in order} | = \frac{15!}{10!}$$

because we select r from n ($0 \leq r \leq n$) in lots of problems... make it a function:

$$P(n, r) = \frac{n!}{(n-r)!} = n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot (n-r+1)$$

(ex) $P(15, 5) = \frac{15!}{10!} = \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot \cancel{10!}}{\cancel{10!}} = \boxed{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11}$

Combinations: arrange n objects: $n!$
but choose (without order) r objects $\rightarrow r!$
so $(n-r)$ have not been chosen $\rightarrow (n-r)!$



b/c we do probs like this a lot ... make it a function

$$C(n, r) = \frac{n!}{r! (n-r)!}$$

notation: $\binom{n}{r} = \frac{n!}{r! (n-r)!}$

ex: 13 people. 9 guys, 4 girls

a) how many ways to pick 5 to play basketball

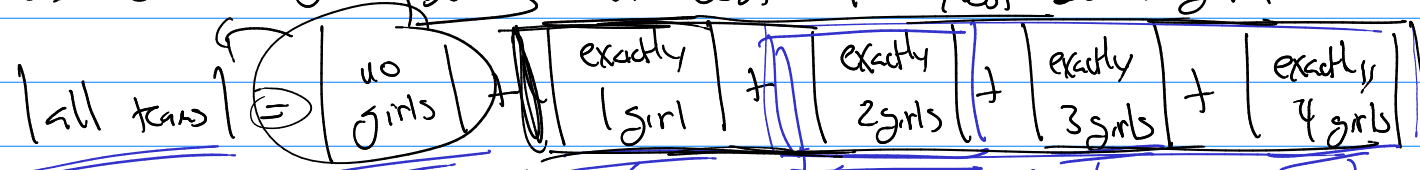
order matters

$$P(13, 5) = \frac{13!}{8!}$$

b) choose 5. don't care about order

$$\binom{13}{5} = \frac{13!}{5! 8!}$$

c) choose 5 but at least 1 must be a girl



$$\binom{13}{5} = \binom{4}{0} \binom{9}{5} + \binom{4}{1} \binom{9}{4} + \binom{4}{2} \binom{9}{3} + \binom{4}{3} \binom{9}{2} + \binom{4}{4} \binom{9}{1}$$

$$\left| \text{at least 1} \right| = \frac{13!}{5! 8!} - \frac{9!}{5! 4!}$$

$$= \frac{4!}{1! 3!} + \frac{9!}{4! 5!} + \frac{4!}{2! 2!} + \frac{9!}{3! 6!} + \frac{4!}{3! 1!} + \frac{9!}{2! 7!} + \frac{4!}{4! 0!} + \frac{9!}{1! 8!}$$