Math 321

Q's n consecutive integers

- $I, I+1, I+2, \ldots, I+n-1$
- any number N divide by n

$$\frac{N}{n} = \text{int} + \frac{r}{n} \rightarrow N = n \cdot q + r \qquad 0 \le r \le n-1$$

$9, 10, 11$ ($q+1$, $q+2$)

ex to play around: - 5 ints $\rightarrow$ $12, 13, 14, 15, 16$
- mod(, 5) $\quad 2, 3, 4, 0, 1$

objects: $I, I+1, I+2, \ldots, I+(n-1)$

boxes: mod([ ] , n)

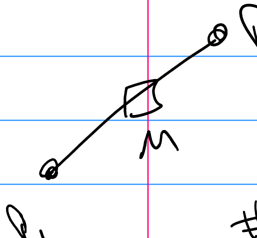
take mod n for each. call $\text{mod}(I, n) = r$

$r, r+1, r+2, \ldots, r+(n-1)$

consecq. remainders: n remainder, n possible remainders
each remainder has exactly one int.

6.2 #10, #11 (2D, 3D)

midpts 2D $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ for $P_1 = (x_1, y_1)$, $P_2 = (x_2, y_2)$



3D $\left(\boxed{\frac{x_1+x_2}{2}}, \boxed{\frac{y_1+y_2}{2}}, \boxed{\frac{z_1+z_2}{2}}\right)$ for $P_1 = (x_1, y_1, z_1)$, $P_2 = (x_2, y_2, z_2)$

#10, #11 ask when midpts are (int, int), (int, int, int)

Means: $\frac{\text{Cord}_1 + \text{Cord}_2}{2} = \text{int} \rightarrow$ Means: $\boxed{\text{Cord}_1 + \text{Cord}_2 = 2 \cdot \text{int}}$ even

$\boxed{6.4}$ Application of $\binom{n}{r}$: Binomial th^{m}

$$(x+2y)^3 = 1\cdot x^3 + 3\cdot x^2(2y)^1 + 3\cdot x^1(2y)^2 + 1\cdot(2y)^3$$

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & 2 & & 1 & \\ 1 & & 3 & & 3 & & 1 \end{array}$$

$$(\square+\Delta)^n = (\square+\Delta)\cdot(\square+\Delta)\cdot \ldots \cdot(\square+\Delta)$$

in words $\underbrace{(\square \text{ or } \Delta) \text{ and } (\square \text{ or } \Delta) \text{ and } \ldots \text{ and } (\square \text{ or } \Delta)}_{n\text{-times}}$

$$= \binom{n}{0}\square^n \text{ or } \binom{n}{1}\square^{n-1}\Delta^1 \text{ or } \binom{n}{2}\square^{n-2}\Delta^2 \text{ or } \ldots \text{ or } \Delta^n$$

$$(\square+\Delta)^n = \binom{n}{0}\square^n + \binom{n}{1}\square^{n-1}\Delta^1 + \ldots + \binom{n}{j}\square^{n-j}\Delta^j + \ldots + \binom{n}{n}\Delta^n$$

$$\boxed{(\square+\Delta)^n = \sum_{j=0}^{n}\binom{n}{j}\square^{n-j}\Delta^j}$$ binomial th^{m}

$$(x+2y)^3 = \frac{3!}{3!\,0!}x^3(2y)^0 + \frac{3!}{2!\,1!}x^2(2y)^1 + \frac{3!}{1!\,2!}x(2y)^2 + \frac{3!}{0!\,3!}x^0(2y)^3$$

$$= x^3 + 3x^2(2y) + 3x(2y)^2 + (2y)^3$$

$$= x^3 + 6x^2y + 12xy^2 + 8y^3$$

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ex $(x - y/2)^{100} \xrightarrow{5^{th}}$ term $\boxed{\frac{100!}{96!\,4!}(x)^{96}(-y/2)^4}$

Pascal's Triangle

$$\binom{0}{0}$$
$$\binom{1}{0} \quad \binom{1}{1}$$
$$\binom{2}{0} \quad \binom{2}{1} \quad \binom{2}{2}$$
$$\binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3}$$
$$\boxed{\binom{4}{0} \quad \binom{4}{1} \quad \binom{4}{2} \quad \binom{4}{3} \quad \binom{4}{4}}$$

1
1 1
1 2 1
1 3 3 1
p 1 4 6 4 1
1 5 10 10 5 1

thm

$$\boxed{\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}}$$

pf (use Algebra)

$$\binom{\square}{\Delta} = \frac{\square!}{\Delta!\,(\square-\Delta)!}$$

$$\frac{(n+1)!}{k!\,(n+1-k)!} \stackrel{?}{=} \frac{k \cdot n!}{k\cdot(k-1)!\,(n-k+1)!} + \frac{n!}{k!\,(n-k)!}\cdot\frac{(n-k+1)}{(n-k+1)}$$

$$\left[\begin{aligned} k! &= k\cdot(k-1)(k-2)\cdots(1) \\ (k-1)! &= (k-1)(k-2)\cdots(1) \end{aligned}\right.$$

$$\left[\begin{aligned} (n-k+1)! &= (n-k+1)(n-k)(n-k-1)\cdots(1) \\ (n-k)! &= (n-k)(n-k-1)\cdots(1) \end{aligned}\right.$$

or use counting proof

have a task that can be counted in two ways.

then $|\text{way}_1| = |\text{way}_2|$

task: you have n students plus the teacher. choose k people.

$$|\text{way}_1| = \binom{n+1}{k}$$

$|\text{ways}| =$ have teacher or do not have teacher

$= (1)\cdot\binom{n}{k-1} + \binom{n}{k}$

get teacher, take $k-1$ from students

take all k from students.

So $\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$

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1 → 1

1 1 → 2

1 2 1 → 4

1 3 3 1 → 8

1 4 6 4 1 → 16

$\binom{4}{0}+\binom{4}{1}+\binom{4}{2}+\binom{4}{3}+\binom{4}{4} = 2^4$

1

1 1 2

1 1 3

1 2 1 5

1 3 3 1 8

1 4 6 4 1

1 5 10 10 5 1