Math 321

Q's

6.3 #11 $\quad b_1 b_2 \dots b_{10} \qquad b_i = \{0,1\}$

$$|\text{all bit strings}| = \underbrace{2\cdot 2\cdot \dots \cdot 2}_{10\text{-bits}} = 2^{10}$$

$$|\text{all}| = \left|\begin{matrix}\text{no}\\ \text{1's}\end{matrix}\right| + \left|\begin{matrix}\text{exactly}\\ \text{one 1}\end{matrix}\right| + \left|\begin{matrix}\text{exactly 2}\\ \text{1's}\end{matrix}\right| + \dots + \left|\begin{matrix}\text{exactly 10}\\ \text{1's}\end{matrix}\right|$$

a) $\left|\begin{matrix}\text{exactly 4}\\ \text{1's}\end{matrix}\right| =$ get 4-1's and of 10 spots choose 4 and fill rest with 0's $= \binom{10}{4}$

$$(1)\cdot\binom{10}{4}\cdot(1) \qquad = \frac{10!}{4!\,6!}$$

$$\left|\begin{matrix}\text{exactly 3}\\ \text{1's}\end{matrix}\right| = \binom{10}{3} = \frac{10!}{3!\,7!}$$

b) $|\text{at most 4 1's}| = \left|\begin{matrix}\text{exactly 0}\\ \text{1's}\end{matrix}\right| + \left|\begin{matrix}\text{exactly 1}\\ \text{1's}\end{matrix}\right| + \dots + \left|\begin{matrix}\text{exactly 4}\\ \text{1's}\end{matrix}\right|$

$$= \binom{10}{0} + \binom{10}{1} + \binom{10}{2} + \binom{10}{3} + \binom{10}{4}$$

$$= \frac{10!}{0!\,10!} + \frac{10!}{1!\,9!} + \dots + \frac{10!}{4!\,6!} = ?$$

ex $\quad |\text{at most 7-1's}| = \binom{10}{0} + \binom{10}{1} + \dots + \binom{10}{7}$

$$= 2^{10} - \binom{10}{8} - \binom{10}{9} - \binom{10}{10}$$

6.4 #11 $\quad \left(x^2 - \frac{1}{x}\right)^{100} = (\text{polynomial like})$

$$(\square + \triangle)^{100} = \frac{100!}{100!\,0!}\square^{100} + \frac{100!}{99!\,1!}\square^{99}\triangle^{1} + \frac{100!}{98!\,2!}\square^{98}\triangle^{2} + \dots$$

$$= x^{200} - \frac{100!}{99!\,1!}x^{197} + \frac{100!}{98!\,2!}(x^2)^{98}(-x^{-1})^{2} - \dots + x^{-100}$$

$$= x^{194}$$

$$= x^{200} + 0\cdot x^{199} + 0\cdot x^{198} - \frac{100!}{99!1!}x^{197} + 0\cdot x^{196} + 0\cdot x^{195} + \frac{100!}{98!2!}x^{194} + \cdots + x^{-100}$$

question: (26) $\binom{?}{?}x^{31}$ $\binom{?}{?}x^{-7}$

$$(\square + \triangle)^k = \sum_{j=0}^{k}\binom{k}{j}\square^{k-j}\triangle^{j}$$

$$\left((x^2)+(-x^{-1})\right)^{100} = \sum_{j=0}^{100}\binom{100}{j}(x^2)^{100-j}(-x^{-1})^{j}$$

$$= \sum_{j=0}^{100}\binom{100}{j}(-1)^j x^{200-3j}$$

wanted some x^k so $k = 200-3j$

$\to j = \frac{200-k}{3}$

terms: $\binom{100}{j}(-1)^j x^{200-3j}$

$$\boxed{\binom{100}{\frac{200-k}{3}}(-1)^{\frac{200-k}{3}} x^k}$$ — only if $3 \mid 200-k$

if $3 \nmid 200-k$ coeff $= 0$

$$\boxed{-\frac{100!}{69!31!}x^{-7}}$$

$3 \overline{)207}$ = 69

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Advanced Counting:

ch8 consider a task that counts n-objects in a problem.

a_1 = # of ways to do task with 1 object
a_2 = # of ways to do task with 2 objects
⋮
a_n = # of ways to do task with n objects

ex a_n = # of ways to choose exactly n - 1's in a bit string of length 100.

$$a_n = \frac{100!}{n!\,(100-n)!} = \binom{100}{n}$$

so $a_n = f(n)$ → closed function
→ open function (Induction or recursive formula)

$\boxed{\text{Why have recurrance relations?}}$

use these when larger numbers of objects are formed from smaller numbers.

ex you have \$1 coins, \$3 coin, \$3 bill, \$5 bill

C_n = # of ways give someone \$n.

recurrence relation:

$$\$n = \binom{\$1}{\text{coin}} \cdot (C_{n-1}) + \overset{\text{give \$3}}{(2)} \cdot C_{n-3} + \overset{\text{give \$5}}{(1)} \cdot C_{n-5}$$

$$\boxed{C_n = C_{n-1} + 2C_{n-3} + C_{n-5}}$$

Basis: $C_0 = 1$