Math 321

Q's 6.4 #31 power set of $|S| = n$

$$|P(S)| = 2^n$$

ex $P(S) = \{$ set of $\emptyset$ elements $\emptyset \leftarrow \binom{n}{0}$

sets of 1 element $\{a_1\}, \{a_2\}, \ldots \{a_n\} \leftarrow \binom{n}{1}$

sets of 2 elements $\{a_1, a_2\}, \{a_1, a_3\}, \ldots \leftarrow \binom{n}{2}$

$\vdots$

all of S $\}$

$$\text{all subsets} = \boxed{\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \ldots + \binom{n}{n}}$$

even element subsets　　odd element subsets

also Corollary 2 on p. 417

$$(x+y)^n = \binom{n}{0}\underset{1}{x^n} + \binom{n}{1}\underset{1}{x^{n-1}}y^1 + \binom{n}{2}x^{n-2}y^2 + \ldots + \binom{n}{n}y^n$$

let $x = 1$, $y = -1$

$$0 = \binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \ldots \pm \binom{n}{n}$$

$$\boxed{\binom{n}{1} + \binom{n}{3} + \ldots = \binom{n}{0} + \binom{n}{2} + \ldots}$$

8.1 / 8.2

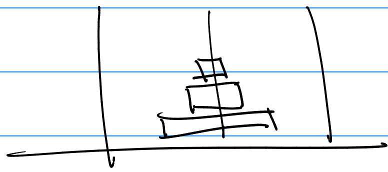
Recurrence Relations

$a_n =$ expression of $a_{n-1}, a_{n-2}, \ldots, a_{n-k}$

ex $f_n = f_{n-1} + f_{n-2}$

$a_n = a_{n-1} + 2a_{n-3} + a_{n-5}$

ex



$H_3 = H_2 + 1 + H_2$

$$\boxed{H_n = 2H_{n-1} + 1}$$

Solve a recurrence relation

↳ given relation → find a closed function.

ex $H_n = 2H_{n-1} + 1$, $H_1 = 1$

$1, 3, 7, 15, 31, \ldots$

$2 \quad 4 \quad 8 \quad 16 \quad 32 \leftarrow 2^n$

Solve? $H_n = f(n)$

① guess and check

$H_n = 2^n - 1$

check: $2^n - 1 \stackrel{?}{=} 2(2^{n-1} - 1) + 1$

$\stackrel{?}{=} 2^n - 2 + 1$

$\stackrel{?}{=} 2^n - 1$ ✓ true.

② forward / backward iteration

see seq's / Series

$\boxed{8.2}$ given a relation of specific form → guess a specific soln.

ex (see video)

$a_n = c_1 a_{n-1} + c_2 a_{n-2} + c_3 a_{n-3} + \cdots + c_k a_{n-k}$

Soln $a_n = r^n$

c_i are constants

⇒ linear homogeneous recurrance relation of degree k with constant coeff.

Soln is $a_n = r^n$ $\boxed{Q}$ how to find r (or r's)?

ex $a_n = 1 \cdot a_{n-1} + 1 a_{n-2}$ $a_n = r^n$

plug in $r^n = r^{n-1} + r^{n-2}$ ⇒ $1 = \frac{1}{r} + \frac{1}{r^2}$

$r^n = \frac{r^n}{r} + \frac{r^n}{r^2}$ $r^2 = r + 1$

⇒ $r^2 - (1)r - (1) = 0$

$r = \frac{1 \pm \sqrt{5}}{2}$ $r = \frac{1+\sqrt{5}}{2}$ $r = \frac{1-\sqrt{5}}{2}$

$a_n = (c_1)\left(\frac{1+\sqrt{5}}{2}\right)^n + (c_2)\left(\frac{1-\sqrt{5}}{2}\right)^n$

use: $a_0 = 0$ $a_1 = 1$

$c_1 + c_2 = 0$

$c_1\left(\frac{1+\sqrt{5}}{2}\right) + c_2\left(\frac{1-\sqrt{5}}{2}\right) = 1$

$$\boxed{a_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n}$$

In general $\quad a_n = c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k}$

Soln: $a_n = r^n$ (find r's)

$$r^k - c_1 r^{k-1} - c_2 r^{k-2} - \cdots - c_k = 0$$

k^{th} degree poly. to solve.

ex $\quad a_n = a_{n-1} + 2a_{n-3} + a_{n-5} = 1\cdot a_{n-1} + 0\cdot a_{n-2} + 2a_{n-3} + 0\cdot a_{n-4} + 1\cdot a_{n-5}$

Solve: $\quad r^5 - 1\cdot r^4 - 0\cdot r^3 - 2\cdot r^2 - 0\cdot r - 1 = 0$

$$r^5 - r^4 - 2r^2 - 1 = 0$$

$r_1 = ?\quad r_2 = ?\quad r_3 = ?\quad r_4 = ?\quad r_5 = ?$

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ex $\quad a_n = \boxed{\text{stuff}}$

characteristic poly.

$$(r-2)(r-2)(r+1)(r+1)(r+1)(r-3) = 0$$

$\underbrace{r_1 = 2 \quad r_2 = 2}\quad \underbrace{r_3 = -1 \quad r_4 = -1 \quad r_5 = -1}\quad \underbrace{r_6 = 3}$

$a_n = (\boxed{})(2)^n$
$+ (\boxed{})(-1)^n$
$+ (\boxed{})(3)^n$

polynomial of n : |terms| = |mult|

$$\boxed{a_n = (a+bn)\,2^n + (c+dn+en^2)(-1)^n + (f)\,3^n}$$