Math 321

Q's

$$\boxed{\begin{array}{l} H_1 = 1 \\ H_n = 2H_{n-1} + 1 \end{array}} \stackrel{?}{=} \boxed{H_n = 2^n - 1} \qquad n = 1, 2, \dots$$

Scratch

$n=1$	$H_1 = 1$	$H_1 = 2^1 - 1 = 1$	Same
$n=2$	$H_2 = 2H_1 + 1 = 3$	$H_2 = 2^2 - 1 = 3$	Same
$\vdots$		$\vdots$	
$n=k$	$H_k = 2H_{k-1} + 1$	$H_k = 2^k - 1$	
$n=k+1$	$H_{k+1} = 2H_k + 1$	$H_{k+1} = 2^{k+1} - 1$	
$\vdots$		$\vdots$	

Pf: **Basis:** show true for 1^{st} case $(n=1)$ Same?

recursive $H_1 = 1$ closed $H_1 = 2^1 - 1 = 1$ $\boxed{\text{true}}$

Inductive: Assume recursive form = closed form (gives same numbers) for $n=1, n=2, \dots, n=k$

Show: it is true for $n = k+1$

(weak) use $n=k$

I.H. $\boxed{H_k = 2H_{k-1} + 1 = 2^k - 1}$

what about $H_{k+1} = 2\boxed{H_k} + 1$ $\qquad$ $H_{k+1} = 2^{k+1} - 1$

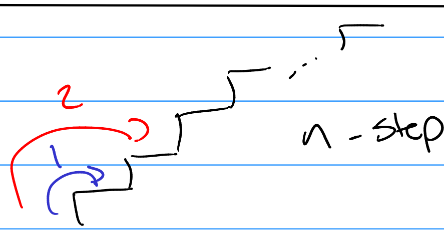
I.H.

$$\begin{aligned} H_{k+1} &= 2(2^k - 1) + 1 \\ &= 2^{k+1} - 2 + 1 \\ &= 2^{k+1} - 1 \end{aligned}$$

Same

So true.

8.1 #11



C_n = number of ways to walk n-steps

How to walk up steps?

C_n = (take 1 step) and (walk the rest $n-1$) or (take 2) and (walk rest $n-2$)

$$C_n = 1 \cdot C_{n-1} + 1 \cdot C_{n-2}$$

$$C_n = C_{n-1} + C_{n-2}$$

$\cancel{C_0 = 1}$

$C_1 = 1$

$C_2 = 2$

(ex) n-steps — 1 step left or right; 2 steps right

$$C_n = (2)\,C_{n-1} + (1)\,C_{n-2}$$

$$\boxed{C_n = 2C_{n-1} + C_{n-2}} \qquad \boxed{\begin{aligned} C_0 &= 1 \\ C_1 &= 2 \end{aligned}}$$

$$\begin{array}{ccccc} C_0 & C_1 & C_2 & C_3 & C_4 \end{array}$$

$$\{C_n\} = 1, 2, 5, 12, 29, \ldots$$

8.2 (Solve) C_n = closed function

(ex) $C_n = r^n$ $\quad r = ?$

Solve: $r^2 - 2r - 1 = 0$

$$r = \frac{2 \pm \sqrt{4+4}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$$

$$r_1 = (1+\sqrt{2}) \quad r_2 = (1-\sqrt{2})$$

$$C_n = (a)(1+\sqrt{2})^n + (b)(1-\sqrt{2})^n \quad \text{b/c} \quad \begin{aligned} C_0 &= 1 \\ C_1 &= 2 \end{aligned}$$

$C_0 = 1 \rightarrow a + b = 1 \rightarrow a = 1 - b$

$C_1 = 2 \rightarrow a(1+\sqrt{2}) + b(1-\sqrt{2}) = 2 \rightarrow (1-b)(1+\sqrt{2}) + b(1-\sqrt{2}) = 2$

$$1+\sqrt{2} - b(1+\sqrt{2}) + b(1-\sqrt{2}) = 2$$

$$1+\sqrt{2} - 2\sqrt{2}\, b = 2$$

$$b = \frac{1-\sqrt{2}}{-2\sqrt{2}} = \frac{\sqrt{2}-1}{2\sqrt{2}} = b$$

$$a = 1-b = \left(1 - \frac{\sqrt{2}-1}{2\sqrt{2}}\right)$$

$$\boxed{a_n = \left(1-\frac{\sqrt{2}-1}{2\sqrt{2}}\right)(1+\sqrt{2})^n + \left(\frac{\sqrt{2}-1}{2\sqrt{2}}\right)(1-\sqrt{2})^n}$$

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ex

$a_n = $ ~~~~

given

$$r^k - \ldots = 0$$

given:

$$(r-2)(r-2)(r-2)(r+3)(r+1)(r+1) = 0$$

$$r_1 = 2 \qquad r_2 = -3 \qquad r_3 = -1$$

$$\boxed{\begin{aligned} a_n = &(a+bn+cn^2)(2)^n \\ &+(\quad d \quad)(-3)^n \\ &+(e+fn)(-1)^n \end{aligned}}$$

Exam $\boxed{11 \text{ problems}}$

6.1 Sum, Product, Overcounts (3 probs)

1. basic sum/product.
2. Inclusion/Exclusion (div. numbers)
3. division

6.2 Pigeonhole (1 prob)

1. generalized version

6.3 $P(n,r)$, $C(n,r)$ (2 probs)

1.
2. pick? choose?

6.4 $(x+y)^n$ (2 probs)

1. use $(x+y)^n =$
2. prove: $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k+1}$

8.1 / 8.2 recurrence relations (3 probs)

1. Find a recurrence relation, initial values, given seq.
2. Solve deg.-2 problem
   ex $\boxed{a_n = -2a_{n-1} + 8a_{n-2}}$ $\boxed{a_0 = 2 \quad a_1 = -2}$
3. given roots → show soln (no initial values)

$a_n = \boxed{}$

$(r-2)(r-2)(r+3) = 0$

ex $(r+2)(r-4)=0$

$a_n = ?$

ex $(r+2)(r+2)(r-4)=0$

$a_n = ?$

ex See earlier