Math 321

Q's

Final 16 probs = 4 per exam

→ "Variations" on exam problems

① could be exact same problem

② Small variation

into: Mark beats a silly kid in a race → Some silly kid beat Mark in a race.

③ Same concept, problem with induction.

Same concept: prove $1+2+\dots+n = \frac{n(n+1)}{2}$ → any induction formula type problem.

Note:

$\cancel{\#}$ ← means "not on the final"

\# ← means "on the final"

Math 321 … Exam 1

? o

1) Construct the truth table everyone should know.

o or 6 these are language → Symbols / Symbols → language

? o

2) a) Express "For the mouse to defeat the cat it is sufficient yet not necessary that the mouse drinks lots of coffee or the cat is sleepy" using propositional symbols and logical operators.

b) Construct the truth table for your compound proposition from part (a).

3) Use a truth table to show that the statements $(p \to q) \land (p \to r)$ and $p \to (q \land r)$ are logically equivalent.

$l \equiv r$

4) Show that $(p \to q)$ and $(\neg q \to \neg p)$ are logically equivalent by discussion.

(1) when is l false?
(2) when is r false?
Compare.

5) Use logical equivalences to show that $(p \land q) \to p$ is a tautology.

$(p \land q) \to p \equiv \neg(p \land q) \lor p \equiv \neg p \lor \neg q \lor p$

6) a) Let $S(u)$ mean that "u is silly," $F(v)$ mean that "v is fast," and $B(a, b)$ mean that "a has beat b in a race", where the universe of discourse for each variable consists of all children. Express $\exists x(S(x) \land \forall y(F(y) \to B(x, y)))$ by a simple English sentence.

$\forall k(F(k) \to [B(k, \text{Mark}) \lor \exists c(S(c) \land B(c, k))])$

b) Use quantifiers and the propositional functions given in part (a) to express "Every fast kid has either beat Mark in a race or been beat by some silly kid in a race".

7) The following argument is not valid. "You do not do every problem in the book or you learn discrete mathematics. You learned discrete mathematics. Therefore, you did every problem in the book." Explain why it isn't valid..

? 6

8) Come up with two valid conclusions for the set of premises: "If I drink coffee at bedtime, then I have strange dreams." "I have strange dreams if there is music playing while I sleep." "I did not have strange dreams." "Having strange dreams is sufficient for me to pass Math 321." Explain your answers.

rational or irrational

9) Prove that $\sqrt{2}$ is irrational. (Include the proof of the needed lemma)

$\sqrt{2}^{\sqrt{2}}$ is a real number.

10) For the integers 2,3,4,... Prove: if $n^2 < 2^n$, then $n > 4$.

Case 1 $\sqrt{2}^{\sqrt{2}}$ is rational (witness)

? o

11) Show that there exist irrational numbers x and y such that, x^y is rational.

Case 2 $\sqrt{2}^{\sqrt{2}}$ is irrational

then $\left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = 2$ (witness)

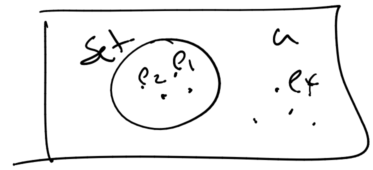
Math 321 … Exam 2

? 6

1) Use set builder notation and roster forms to represent each of the following sets. The set A is even integers from 2 to 9, the set B is all integers that are a multiple of 3 from -5 to 7, and among a universe of discourse of integers from -6 to 10. And then

Set = {a_1, a_2, ..., a_n}
Set = {e | P(e)}

illustrate all the sets and the universe of discourse with a single Venn Diagram.



? 2) For $A = \{s, c\}$ and $B = \{3\}$ find $P(B \times A)$.

$B \times A = \{\underbrace{(3,s)}_{e_1}, \underbrace{(3,c)}_{e_2}\}$

? 3) Represent $A \cap \overline{(A \cap B)}$ with a Venn Diagram by using a membership table.

4) Show that $(A - B) - C = (A - C) - (B - C)$ using set builder notation and logical equivalences.

? 5) Show that if f and g are one-to-one, then $f \circ g$ is also one-to-one.

1-1: $\boxed{f(a) = f(b) \to a = b}$

$f(g(a)) = f(g(b))$

? 6) a) Find a function $f : \mathbb{Z}^+ \to \mathbb{Z}$ where f is not one-to-one and is onto.

b) Find a function $f : \mathbb{Z} \to \mathbb{Z}^+$ where f is one-to-one and is onto.

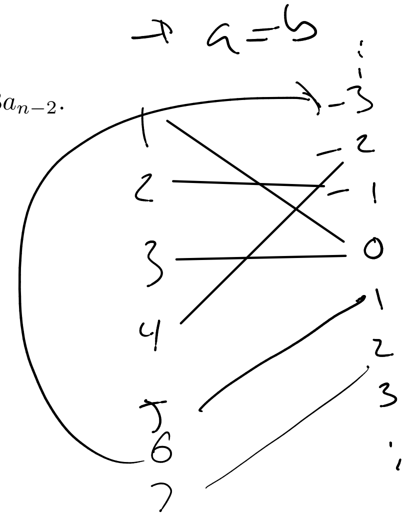
f is 1-1 $\to$ $g(a) = g(b)$

$\to a = b$

? 7) Sequences ...

a) List the first 5 terms of the sequence $a_0 = -1, a_1 = 2$ and $a_n = 2a_{n-1} + 3a_{n-2}$.

b) Find formulae for the sequence: 2, 5, 10, 17, 26, ...



8) Find the value of the sum ...

$$\sum_{k=3}^{9} k^3$$

9) Use the properties of telescoping series to find the value of the sum ...

$$\sum_{k=1}^{21} (k+1)^2 - k^2$$

10) Prove that $\mathbb{Q}$ is countable.

one will be on test!

11) Prove that $\mathbb{R}$ is uncountable.

Math 321 ... Exam 3

? 1) Given $a, b,$ and c are integers, Show that if $a|b$ and $a|c$, then $a|2b - 3c$.

? 2) a) Find $-22 \operatorname{div} 6$ and $-22 \bmod 6$

b) Find 22 div 6 and 22 mod 6

c) List two negative integers and two positive integers that are congruent to -3 modulo 5.

3) Perform the requested operations ...

a) $(1,2,3)_6 + (4,5)_6$ using only base 6 numbers. Write your answer in both base 6 and base 10.

b) $(1,4)_6 \times (2,5)_6$ using only base 6 numbers. Write your answer in both base 6 and base 10.

4) Prove there are infinitely many primes.

5) Find the prime factors, the gcd, and the lcm of 140 and 75 using prime factorization. Don't multiply out the product of primes for your answers.

6) Find the gcd of 140 and 75 using Euclid's Algorithm. Then write the answer as $gcd(140, 75) = s * 140 + t * 75$ for integers s and t.

7) Given the affine-shift function: $f(p) = (7p + 3) \bmod 13$ find the decryption function $f^{-1}(c)$.

8) Given the key of $e = 5$ and $n = 221$ of a public key encryption. Find the encryption function $f(p)$ and decryption function $f^{-1}(c)$.

9) Prove that $1/2 + 1/4 + 1/8 + \ldots + 1/2^n = 1 - 1/2^n$ for $n = 1, 2, 3, \ldots$ using weak induction.

10) Prove all integers $n \geq 2$ are prime or can be written as a product of primes using strong induction.

11) Prove that $f_2 + f_4 + \ldots + f_{2n} = -1 + f_{2n+1}$ when n is a positive integer.

Math 321 ... Exam 4

1) How many student ID's can be made where an ID uses either two digits followed by four uppercase English letters or an ID uses four digits followed by two uppercase English letters or an ID uses six digits? (Do not simplify your answer. Leave it as a product and/or sum of numbers.)

2) Given the integers from 13 to 1032 (including 13 and 1032) how many of them are divisible by 2? How many are divisible by 3? How many are divisible by 2 and 3? How many are divisible by 2 or 3?

$\frac{P(10,4)}{4\cdot 2} = \frac{10!}{6!\cdot 4\cdot 2}$ Sit 10 around a table Same people on left/right = same.

3) How many ways are there to seat four people from ~~a~~ group of ten ~~around~~ a circular ~~table where two~~ seatings are considered the same when everyone has the same immediate left and immediate right neighbor?

$\boxed{\frac{10!}{10\cdot 2}}$

P_{10} P_1 P_2

pigeonhole principle

4) A company stores products in a large warehouse. Storage bins in the warhouse ~~are specified by their~~ aisle, location in the aisle, and shelf. There are 200 aisles, 300 horizontal locations in each aisle, and 10 shelves throughout the warehouse. What is the least number of products the company can have so that at least four products must be stored in the same bin?

5) (Please leave your answers in factorial notation) 9 people (5 Math majors and 4 CS majors) show up for a basketball game.

a) How many ways are there to choose 5 players to play?

b) How many ways are there to pick 5 players to play?

c) How many ways are there to choose 5 players to play if at least two players must be a Math major?

& the exam on blackboard

6) How many committees of five people chosen from 15 people (9 Math faculty and 6 CS faculty) have ~~more more~~ Math faculty committee members than CS faculty members?

+ bit strings of length 10 with a) #0's = #1's etc

7) What is the 22nd term for $(x^2 + x^{-3})^{25}$? Leave your coefficient in factorial notation, but combine the variables together to get a single x to a specific power.

8) Prove $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$ by using a Combinatorial Proof. (not like book)

9) Find a recurrence relation with initial conditions for the number of ways to walk up stairs with n-steps if you can take one step using either or your right leg or left leg. Or you could go up three steps with one large jump. After you have the basis values and recurrence relation write the first 5 values of the sequence.

10) Solve $a_n = a_{n-1} + 6a_{n-2}$ with initial conditions $a_0 = 5$ and $a_1 = 0$.

11) Solve $a_n = -3a_{n-1} + 9a_{n-2} + 27a_{n-3}$.

a) $(r+1)(r+2)(r-3) = 0 \rightarrow a_n$

b) $(r+2)^3(r-3)^2(r+1) = 0 \rightarrow a_n = (a + bn + cn^2)(-2)^n + (d + en)(3)^n + f(-1)^n$