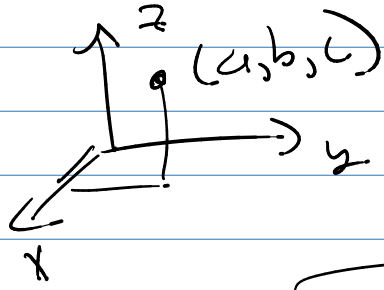
Math 243

Ch 12 3D coord. (+) Vectors

an "Algebra" | objects (+) rules | ← satisfy some properties

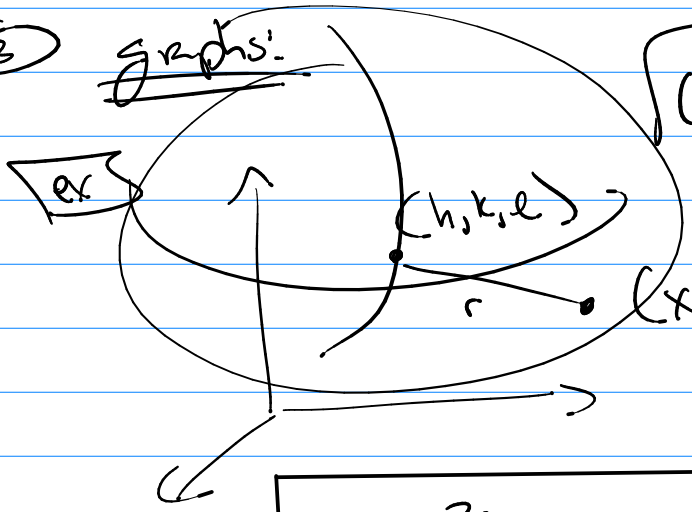
3D ① locations



② distance **3D** $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

③ graphs:

ex



$\sqrt{(x-h)^2+(y-k)^2+(z-l)^2} = r$

$r \geq 0$

$$(x-h)^2+(y-k)^2+(z-l)^2 = r^2$$

Vectors (+) operations, equality?

↑ toys ↑ rules

Vectors: $\vec{v}_1$ or $\mathbf{v}_1$ or $\vec{v}_2$ or $\mathbf{v}_2$

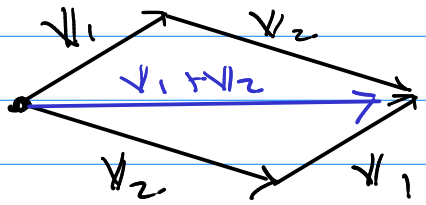
equal

① equal? Same mag. Same direction.

$\mathbf{v}_1 = \mathbf{v}_2$

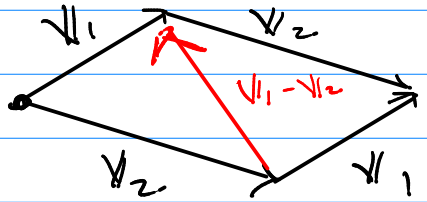
$\mathbf{v}_2 = 3$

② $\mathbf{v}_1 + \mathbf{v}_2$

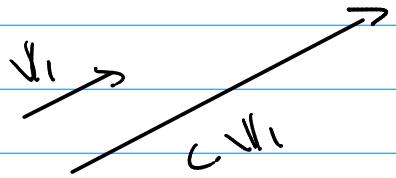


$\mathbf{v}_1 + \mathbf{v}_2 = \mathbf{v}_2 + \mathbf{v}_1$

③ $\mathbf{v}_1 - \mathbf{v}_2$



④ $c\mathbf{v}_1$



Scalar multiplication

Stretch $\mathbf{v}_1$ by $|c|$

if $c > 0$ same direction

$c < 0$ switch direction

⑤ Identity (do nothing under operator)

Inverse (undo under operator)

Vector Addition: Identity $\vec{0}$, 0 zero vector

$\mathbf{v} + 0 = \mathbf{v}$

Inverse: $\vec{v} + (-\vec{v}) = \vec{0}$

why?

$\vec{x} + \vec{v} = \vec{w}$

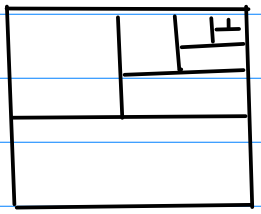
$\vec{x} + \vec{v} + (-\vec{v}) = \vec{w} + (-\vec{v})$

$\vec{x} = \vec{w} - \vec{v}$

Scalar multi.: $(1)\vec{v} = \vec{v}$

$0 \cdot \vec{v} = \vec{0}$

$\frac{1}{2} + \frac{2}{4} + \frac{2}{8} + \frac{2}{16} + \frac{1}{32} + \ldots = 1$



geometric work is "hard"

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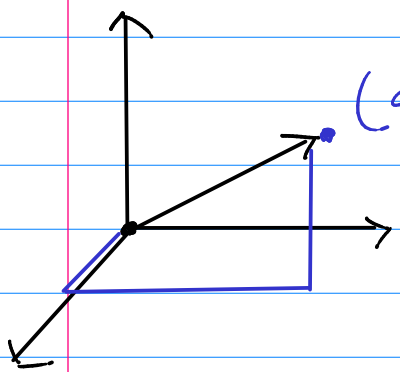
use scalars for vectors



$\vec{v} = \langle a, b, c \rangle$ position vector

$\overrightarrow{P_1P_2} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$

$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

$\vec{a} = \langle a_1, a_2, a_3 \rangle$

$\vec{b} = \langle b_1, b_2, b_3 \rangle$

$\vec{a} + \vec{b} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$

$\vec{a} - \vec{b} = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle$

$$c\vec{a} = \langle ca_1, ca_2, ca_3 \rangle$$