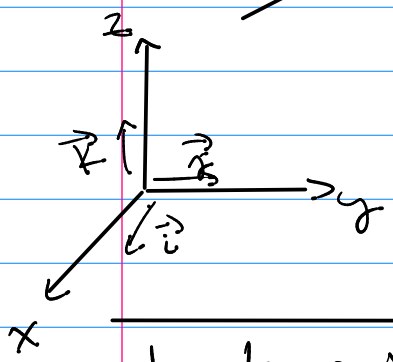
Math 243

Blackboard → Syllabus & Calendar

Backups of Lectures → chaos.math.wichita.edu

Vectors: $\vec{v} = \langle a_1, a_2, a_3 \rangle$



$\vec{i} = \langle 1,0,0 \rangle \quad \vec{j} = \langle 0,1,0 \rangle \quad \vec{k} = \langle 0,0,1 \rangle$

$\vec{v} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$

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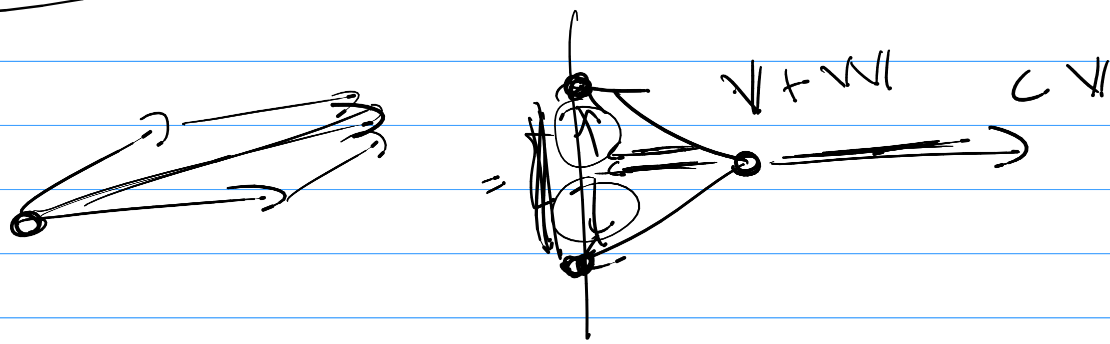
laws/properties of $\vec{v}_1 + \vec{v}_2$ or $c\vec{v}_1$

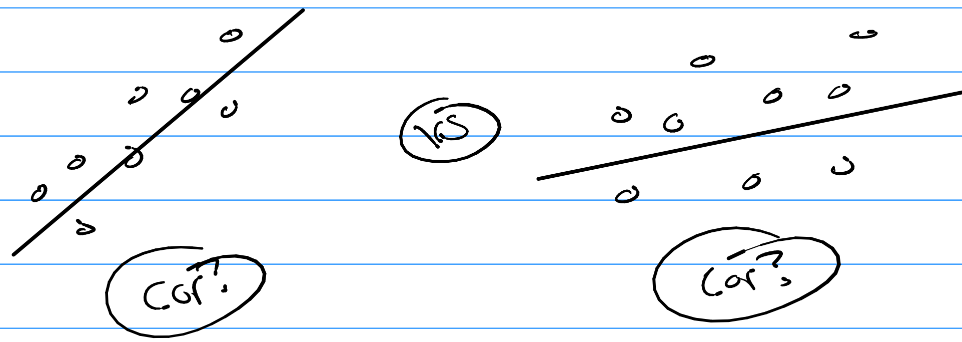
1. $\vec{a} + \vec{b} = \vec{b} + \vec{a}$
2. $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$
3. $\vec{a} + \vec{0} = \vec{a}$, $0 \cdot \vec{a} = \vec{0}$
4. $\vec{a} + (-\vec{a}) = \vec{0}$
5. $c(\vec{a} + \vec{b}) = c\vec{a} + c\vec{b}$
6. $(c+d)\vec{a} = c\vec{a} + d\vec{a}$
7. $(cd)\vec{a} = c(d\vec{a}) = d(c\vec{a})$
8. $1 \cdot \vec{a} = \vec{a}$

Products? Vector (product) Vector = ?

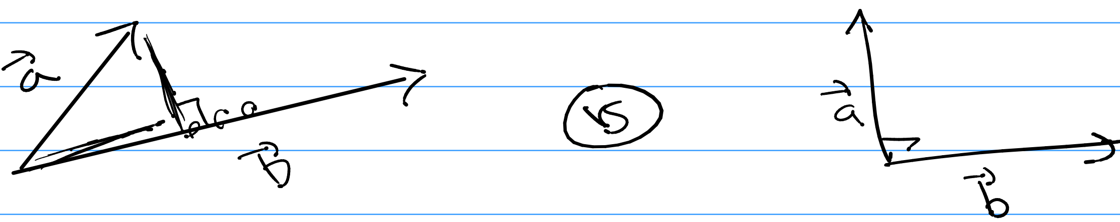


Metrics

Stats ex



vs



Dot product $\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

vector • vector = scalar

Properties

① $\vec{a} \cdot \vec{a} = |\vec{a}|^2$ or $|\vec{a}| = (\vec{a} \cdot \vec{a})^{1/2}$

② $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

③ $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$

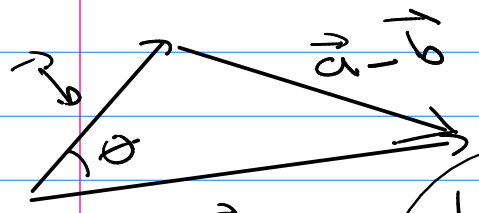
$(\vec{a} + \vec{b}) \cdot \vec{c} = \vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{c}$

④ $(c\vec{a}) \cdot \vec{b} = c(\vec{a} \cdot \vec{b}) = \vec{a} \cdot (c\vec{b})$

~~$a\vec{V} + c$~~

~~vector + scalar~~

(5) $\vec{0} \cdot \vec{a} = 0$



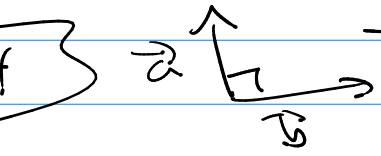
$|\vec{v}|^2 = \vec{v} \cdot \vec{v}$

Law of cosines

$$|\vec{a}-\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\theta$$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} \quad \text{or} \quad \vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

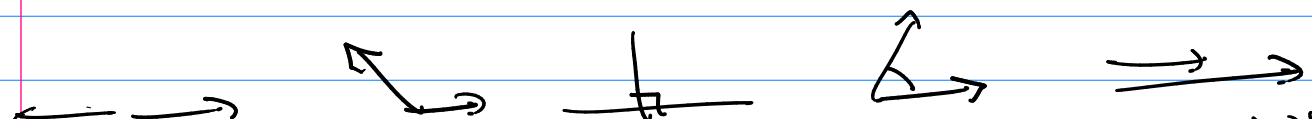
Property: $\vec{a} \perp \vec{b} \rightarrow \vec{a}\cdot\vec{b} = 0$



(6) $\rightarrow\rightarrow$ $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|$

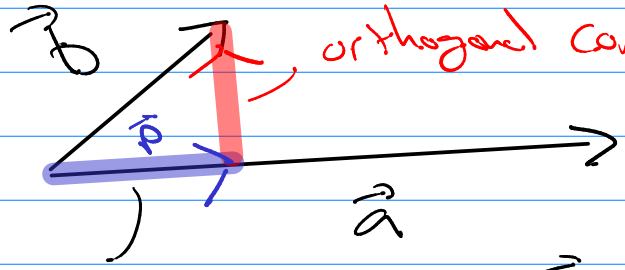
$\leftarrow\rightarrow$ $\vec{a}\cdot\vec{b} = -|\vec{a}||\vec{b}|$

so $-|\vec{a}||\vec{b}| \leq \vec{a}\cdot\vec{b} \leq |\vec{a}||\vec{b}|$



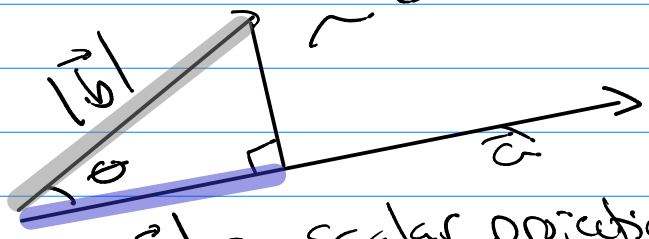
application of $\vec{a} \cdot \vec{b}$

1. Orthogonal test
2. projection (not orthogonal)



orthogonal component of $\vec{b}$ to $\vec{a} = \vec{b} - \vec{p}$

Vector projection of $\vec{b}$ onto $\vec{a} = \vec{p}$



$$\cos\theta = \frac{|\vec{p}|}{|\vec{b}|} \rightsquigarrow |\vec{p}| = |\vec{b}|\cos\theta$$

$|\vec{p}|$ ~ scalar projection of $\vec{b}$ onto $\vec{a}$

$$\text{So } |\vec{p}| = |\vec{b}|\cos\theta = |\vec{b}|\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

$$\boxed{|\vec{p}| = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}}$$

$$\vec{p} = |\vec{p}|\left(\frac{\vec{a}}{|\vec{a}|}\right) = \boxed{\frac{\vec{a}\cdot\vec{b}}{\vec{a}\cdot\vec{a}}\,\vec{a}}$$

unit vector.