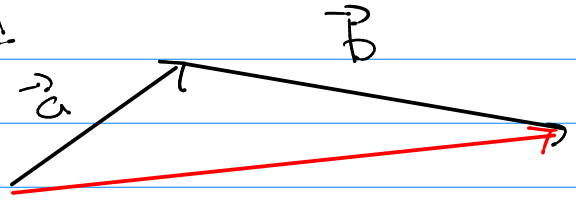
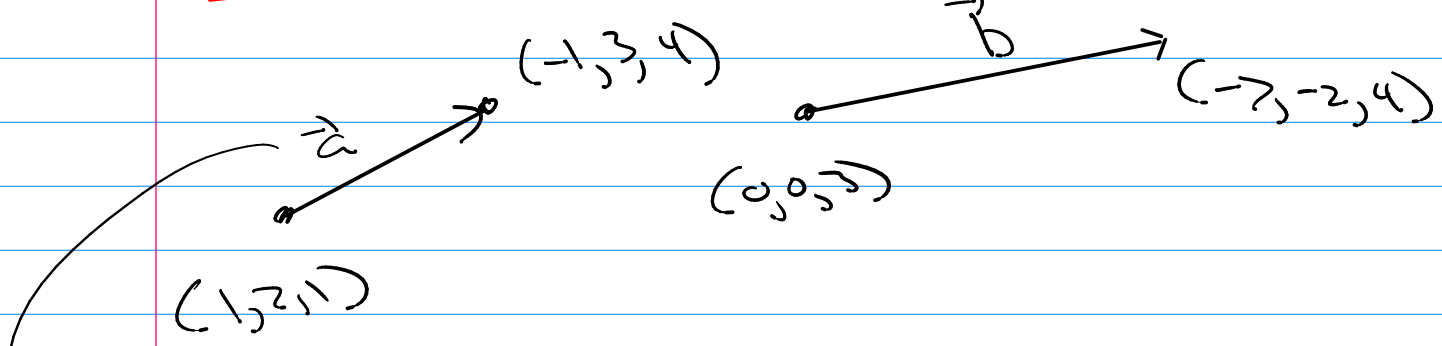
Math 243

Q's!





$\vec{a} = \langle -2, 1, 3 \rangle$ $\quad$ $\vec{b} = \langle -7, -2, 1 \rangle$

$\vec{a} + \vec{b} = \langle -9, -1, 4 \rangle$

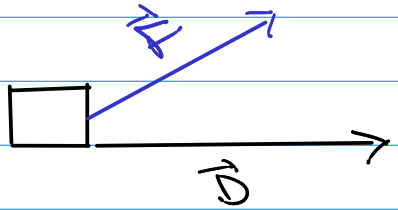
$\vec{a} \cdot \vec{b} = (14) + (-2) + (3) = 15$

$\vec{a} \cdot \vec{b} = |a||b| \cos\theta \rightarrow \theta = \cos^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}\right)$

1) $\theta = \cos^{-1}\left(\frac{15}{\sqrt{14}\sqrt{54}}\right)$

App $\quad$ Work

$W = \vec{F} \cdot \vec{D}$ = Force Vector · Displacement Vector

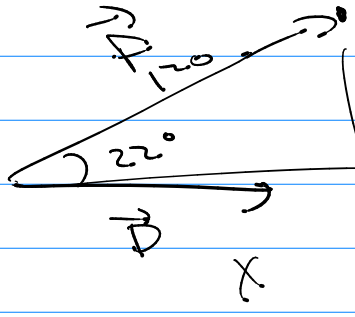


$|W| = |\vec{F}||\vec{D}| \cos\theta$

Ex 2D pull $|\vec{F}| = 120\text{lb}$

move box = 20 ft

angle of 22°



$|w| = 120 \cdot 20 \cos(22°)$

$\vec{b} = \langle 120\cos 22°, 120 \sin 22° \rangle$

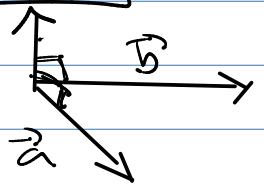
Today: Vectors

Refs: equality, ops: $\vec{a}+\vec{b}$, $c\vec{a}$ (properties)

$\vec{a}\cdot\vec{b}$ (properties)

New Product $\vec{a}\times\vec{b}$ is $(\vec{a}\times\vec{b}) \perp \vec{a}$

$(\vec{a}\times\vec{b}) \perp \vec{b}$



Finding $\vec{a}\times\vec{B}$

① Determinant $|a| = a$

a_{ij}

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}$$

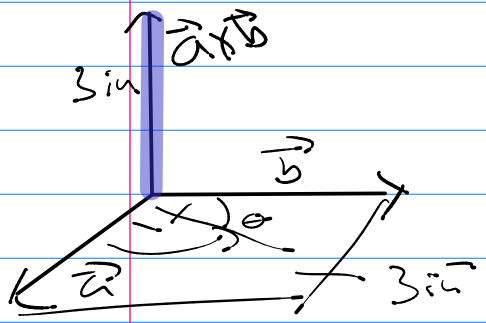
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \langle a_2b_3 - b_2a_3,\ b_1a_3 - a_1b_3,\ a_1b_2 - b_1a_2 \rangle$$

(1st, 2nd, 3rd)

Properties

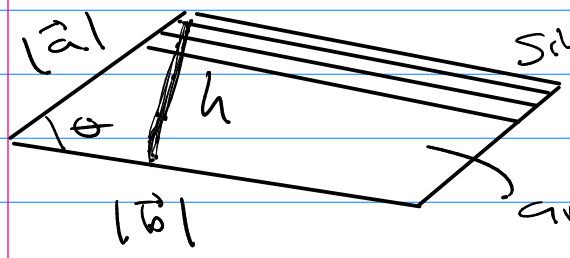
$$|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$$



So if $|\vec{a}\times\vec{b}| = 0$

or $\vec{a}\times\vec{b} = \vec{0}$

→ angle between $\vec{a}, \vec{b}$ is 0. (parallel)



$\sin\theta = \frac{h}{|\vec{a}|}$

area $= |\vec{b}| \cdot h$

area $= |\vec{a}||\vec{b}|\sin\theta$

Properties

① $\vec{a}\times\vec{b} = -(\vec{b}\times\vec{a})$ (not commutative)

② $(c\vec{a})\times\vec{b} = c(\vec{a}\times\vec{b}) = \vec{a}\times(c\vec{b})$

③ $\vec{a}\times(\vec{b}+\vec{c}) = \vec{a}\times\vec{b} + \vec{a}\times\vec{c}$

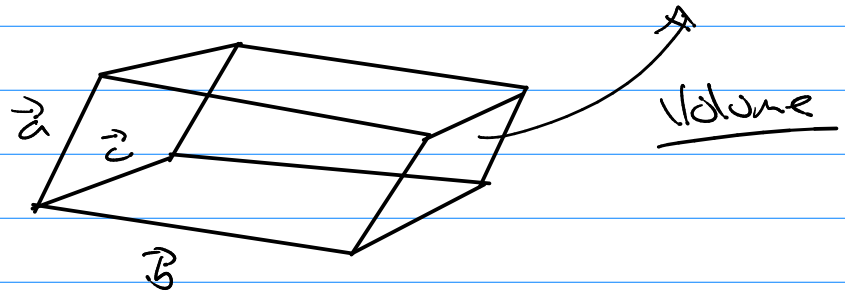
(4) $(\vec{a}+\vec{b})\times\vec{c} = (\vec{a}\times\vec{c}) + (\vec{b}\times\vec{c})$

(5) $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$

(6) $\vec{a}\cdot(\vec{b}\times\vec{c}) = (\vec{a}\times\vec{b})\cdot\vec{c}$

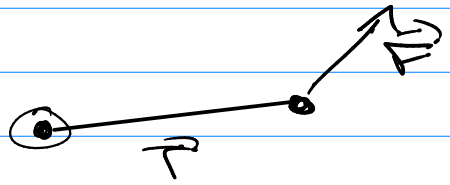
Pf: $\vec{a}\cdot(\vec{b}\times\vec{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

$|\vec{a}\cdot(\vec{b}\times\vec{c})| = |\vec{b}\times\vec{c}|\,|\vec{a}|\,|\cos\theta|$



Apps

Torque



$|\vec{r}\times\vec{F}| = |\vec{r}|\,|\vec{F}|\sin\theta$

$\vec{\tau} = \vec{r}\times\vec{F}$