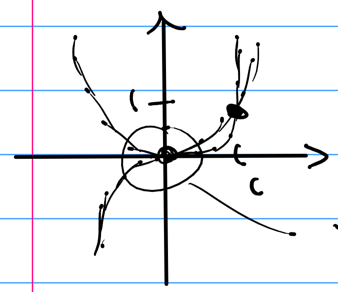
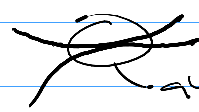
# Math 243

**Q5** 12.3 #31 $\quad f: y = x^2 \quad g: y = x^3$ angles between intersections

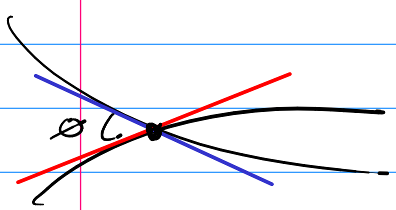


$x^2 = x^3 \rightarrow x^3 - x^2 = 0$

$x^2(x-1) = 0 \rightarrow x=0, x=0, x=1$



angle between tangent lines slope



$f(x)=x^2 \quad g(x)=x^3$

$f'(x)=2x \quad g'(x)=3x^2$

Intersection:

$x=0$

slope of $f$ @ $x=0$

$= 0 \qquad \vec{v}_1 = \langle 1,0 \rangle$

slope of $g$ @ $x=0$

$= 0 \qquad \vec{v}_2 = \langle 1,0 \rangle$

$$\boxed{\theta = 0}$$

$x=1$

slope of $f$ @ $x=1$

$= \frac{2}{1}$

slope of $g$ @ $x=1$

$= 3 = \frac{3}{1}$

$\vec{v}_1 = \langle 1,2 \rangle$

$\vec{v}_2 = \langle 1,3 \rangle$

angle between $\langle 1,2 \rangle, \langle 1,3 \rangle$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} \rightsquigarrow \boxed{\text{on this}}$$

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## Derivatives (Review)

$$\boxed{f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}}$$

Rules: sum, diff, prod, quotient, composition

$$D_x\left[\frac{f}{g}\right] = \frac{f'g - fg'}{g^2}$$

$$D_x\left[f(g(x))\right] = f'(g(x)) \cdot g'(x)$$

Function deriv:

$$D_x[x^n], \quad D_x[\text{trig}]$$

$$D_x[e^x] = e^x \qquad \boxed{D_x[\ln|x|] = \frac{1}{x}}$$

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$$D_x\left[\sin(\cos(x^2)) + e^{x^3 - 2/x}\right] \qquad -2x^{-1}$$

$$= \cos(\cos(x^2))(-\sin(x^2))(2x) + \left(e^{x^3-2/x}\right)\left(3x^2 + \frac{2}{x^2}\right)$$

$$= \boxed{-2x\cos(\cos(x^2))\sin(x^2) + \left(3x^2 + \frac{2}{x^2}\right)e^{x^3 - 2/x}}$$

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Integration

$$\int_a^b f(x)\,dx = \left[\int f(x)\,dx\right]\Big|_a^b$$

(1) You recognize the inside of $\int(\text{inside})\,dx$ as a known result of a derivative.

$$\int \sin(x)\,dx = -\cos x + C$$

(2) after basic algebra you then recognize it.

(ex) $$\int \frac{x^3 - x + 1}{\sqrt{x}}\,dx = \int \left(x^{5/2} - x^{1/2} + x^{-1/2}\right)dx$$

$$= \frac{2}{7}x^{7/2} - \frac{2}{3}x^{3/2} + 2x^{1/2} + C$$

(3) Substitution (inside = result of a chain rule)

$$D_x\left[f(g(x))\right] = f'(g(x))\,g'(x)$$

$$\int f'(g(x))\,g'(x)\,dx = f(g(x)) + C$$

Substitution

let $u = g(x)$
$du = g'(x)\,dx$

$$\int f'(u)\,du = f(u) + C$$
$$= f(g(x)) + C$$

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(ex) $$\int \frac{(\ln(x))^3}{x}\,dx = \int (\ln(x))^3 \left(\frac{1}{x}\right)dx$$

let $u = \ln x$
$du = \frac{1}{x}\,dx$

$$= \int u^3\,du = \frac{1}{4}u^4 + C$$

$$= \boxed{\frac{1}{4}(\ln(x))^4 + C}$$

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$$\int x^2\sqrt{x-1}\,dx = \int x^2\sqrt{u}\,du = \int (u+1)^2\sqrt{u}\,du$$

let $\boxed{u = x-1} \to u+1 = x$

$du = dx$

$$= \int (u^2+2u+1)u^{1/2}\,du$$

$$= \int \left(u^{5/2}+2u^{3/2}+u^{1/2}\right)du$$

$= \text{Finish}$