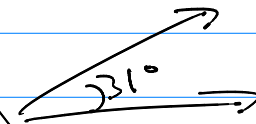
# Math 243

**Q's**

$D_x[f^{-1}(x)] = \frac{1}{f'(f^{-1}(x))}$

$\vec{a}\cdot\vec{b}$ if $|\vec{a}| = 2$, $|\vec{b}| = 4$, $\theta = 31°$




$\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

ex $\vec{a}\cdot\vec{b} = 8\cos 31°$

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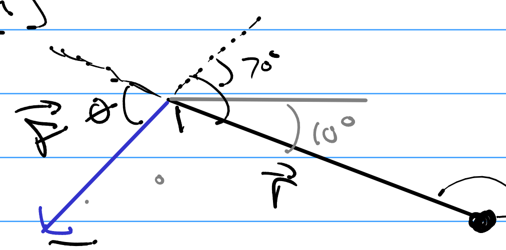
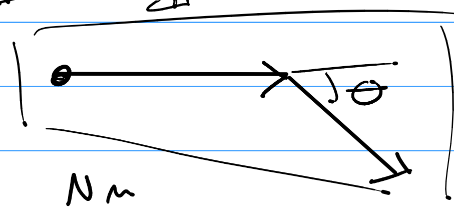
**12.4 #39**




$|\vec{F}| = 60\,N$

$|\vec{r}| = 18\,cm = .18\,m$

$\vec{\tau} = \vec{r}\times\vec{F}$




$|\vec{\tau}| = |\vec{F}||\vec{r}|\sin\theta$

$= (60)(.18)\sin(80°)\ Nm$

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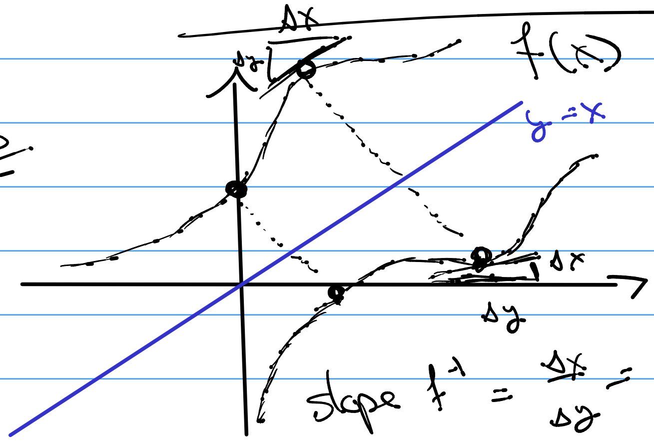
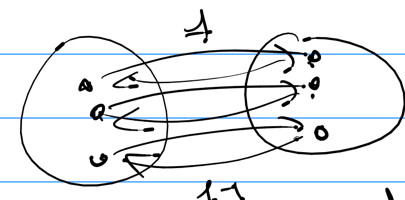
**Ch 6** | 6.1 to 6.4 in Calc 1 |

**Inverse Functions**




slope $f^{-1} = \frac{\Delta x}{\Delta y} = \frac{1}{\Delta y/\Delta x}$

$f(x)$ is one-to-one and onto.




$f^{-1}$ exists

$y = f(x)$

$\boxed{x = f(y)}$

$y = f^{-1}(x)$

so $\text{Slope } f^{-1} = \dfrac{1}{\text{Slope } f}$

$f$ is a bijection

$\boxed{\text{Summary}}$ $y = f(x)$ is an invertable function

$\rightarrow$ find $f^{-1}$ $\quad x = f(y)$

algebra? — yes (explicit) / no (implicit)

$y = f^{-1}(x)$

and $D_x\{f^{-1}(x)\} = \dfrac{1}{f'(f^{-1}(x))}$

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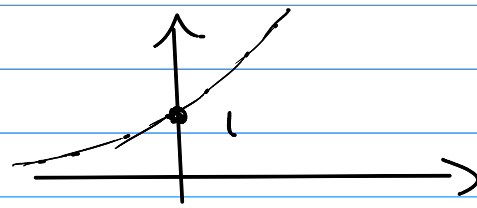
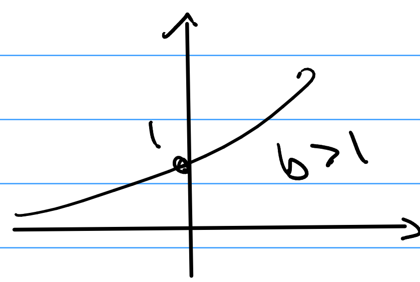
Start of Ch 6 (exponential functions)

$f(x) = e^x$




$f(x) = b^x$





these are invertible so $f^{-1}$ exists

$f(x) = e^x$ $\xrightarrow{\text{Invert}}$ $x = e^y$

can't create an explicit alge. expression

$y = \ln(x)$

Find $y$!

$y = e^x \xrightarrow{\text{Invert}} y = \ln(x)$ means $e^y = x$

$y = b^x \xrightarrow{\text{Invert}} y = \log_b(x)$ means $b^y = x$

$\log_2 8 = 3$ means $2^3 = 8$

$\log_4 8 = 3/2$ means $4^{3/2} = 8$

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Properties & Derivatives

① $b^{\log_b x} = x \qquad \log_b b^x = x$

② $e^{\ln x} = x \qquad \ln e^x = x$

③ $\ln(a \cdot b) = \ln(a) + \ln(b)$

$e^a \cdot e^b = e^{a+b}$

④ $\ln(a^n) = n \ln(a)$

⑤ $\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$

$D_x[e^x] = e^x \qquad D_x[b^x] = (\ln b) b^x$

$D_x[\ln|x|] = \frac{1}{x} \qquad D_x[\log_b|x|] = \left(\frac{1}{\ln b}\right)\frac{1}{x}$

change of base $\log_b x = \frac{1}{\ln b} \ln x$

$$b^x = e^{x(\ln b)}$$

b/c $b^x = e^{\ln(b^x)} = e^{x(\ln b)}$

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Applications 6.5

Natural Growth / Decay Problems

law of natural growth $\quad \frac{dA}{dt} \propto A$

$$\frac{dA}{dt} = kA$$

k: ① constant of proportionality
② growth / decay constant

$\frac{dA/dt}{A} = k \rightarrow$ ③ relative growth rate is a constant

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$\frac{dA}{dt} \boxed{=} kA$

Differential Equation

b/c expressions include a derivative

Soln: is a function

Guess $A = Ce^{kt}$ ($C, k$ are constants)

check: $\frac{dA}{dt} = Ce^{kt} \cdot k = kCe^{kt} = kA$

All natural growth/decay problems ..

$$\frac{dA}{dt} = kA \longrightarrow \text{Soln.} \quad \boxed{A = Ce^{kt}}$$

$k$ : growth rate

$C$ : initial value $\rightarrow$ @ $t=0$ $A(0) = A_0 = C$

Initial Value problem $\boxed{A = A_0 e^{kt}}$

$A_0 = A(0)$

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Probs

① Population: $\boxed{P = P_0 e^{kt}}$ use? know $P_0, k$

② Decay (radioactive decay)

↳ half-life in a fixed time

$A \rightarrow \frac{1}{2} A$

ex

$$A = A_0 e^{kt}$$

in 1hr $\frac{1}{2}$ material is left

$$\frac{1}{2}\cancel{A_0} = \cancel{A_0} e^{k(1)} \rightarrow \frac{1}{2} = e^{k}$$

$$\ln(1/2) = \ln(e^{k})$$

$$\ln(1/2) = k$$

$$\boxed{A = A_0 e^{\ln(1/2)\, t}}$$