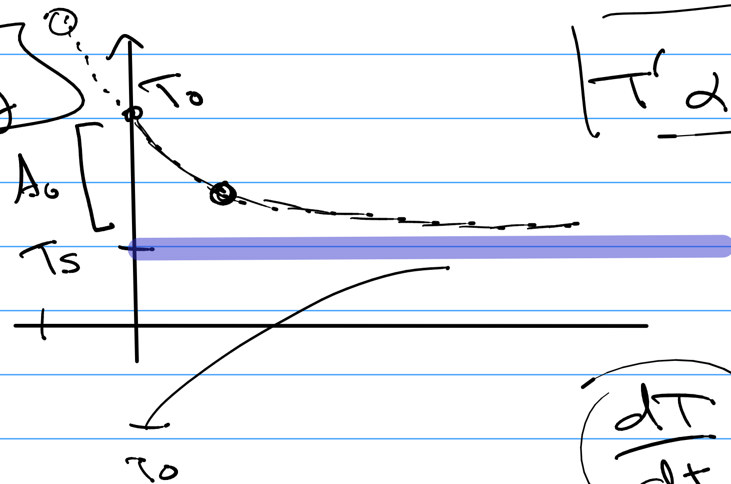
Math 243

Application of $f(x)=e^x$ $\qquad A' \propto A \to \boxed{\frac{dA}{dt}=kA}$

$$A' \propto A \rightsquigarrow A(t)=A_0e^{kt}$$

Cooling



$$\boxed{T' \propto (T-T_s)}$$

$$\frac{dT}{dt}=K\overbrace{(T-T_s)}^{A}$$

Let $A=T-T_s \to \frac{dA}{dt}=\frac{dT}{dt}$

Substitute: $\frac{dA}{dt}=kA \rightsquigarrow A(t)=A_0e^{kt}$

$$A=\boxed{T-T_s} \to \underline{A_0}=\boxed{T_0-T_s}$$

back sub $\quad T-T_s=(T_0-T_s)e^{kt}$

$$\boxed{T=T_s+(T_0-T_s)e^{kt}}$$

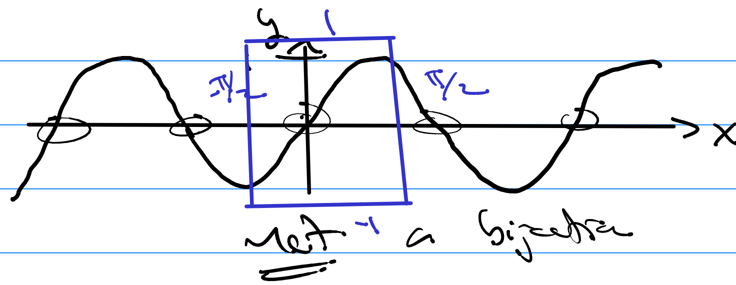
More Inverse Functions (6.6)

$y = f(x)$ if f is a bijection $\to$ f^{-1} exists

Find? $x = f(y) \rightsquigarrow y = f^{-1}(x)$

algebra?

Consider $y = \sin(x)$



Not⁻¹ a bijection

but $y = \sin(x)$ in $-\pi/2 \le x \le \pi/2$, $-1 \le y \le 1$ is bijection

$\to$ f^{-1} exists

Invert

$x = \sin(y) \rightsquigarrow y = \boxed{\arcsin(x)} = \sin^{-1}(x)$

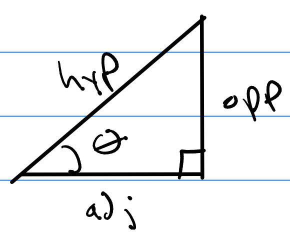
$-1 \le x \le 1 \quad -\pi/2 \le y \le \pi/2$

Note: $\sin(\arcsin(x)) = x, \quad -1 \le x \le 1$

$\arcsin(\sin(x)) = x$

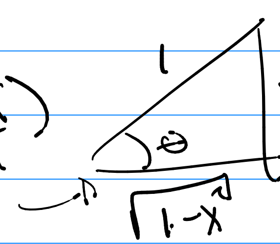
Trig

Note:



$\tan\theta = \dfrac{opp}{adj} \qquad \sin\theta = \dfrac{opp}{hyp}$

$\theta = \arcsin\left(\dfrac{x}{1}\right)$



So

$$\boxed{\tan(\arcsin(x)) = \frac{x}{\sqrt{1-x^2}}}$$

Derivative: $y = \arcsin(x)$ $\qquad \frac{dy}{dx} = ?$

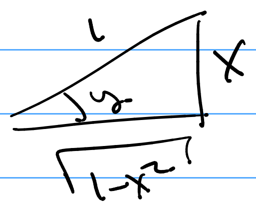
means:

$\sin(y) = x$

Implicit Deriv. $\qquad \cos(y) \cdot \frac{dy}{dx} = 1$

$$\to \frac{dy}{dx} = \frac{1}{\cos(y)} = \frac{1}{\cos(\arcsin(x))} = \frac{1}{\sqrt{1-x^2}}$$

$y = \arcsin(x)$



$\cos(\arcsin(x)) = \sqrt{1-x^2}$

$\sin(y) = x$

$y \stackrel{?}{=} f^{-1}(x)$

$y = \sin(x)$ is invertible if $-\pi/2 \le x \le \pi/2$ $\quad -1 \le y \le 1$

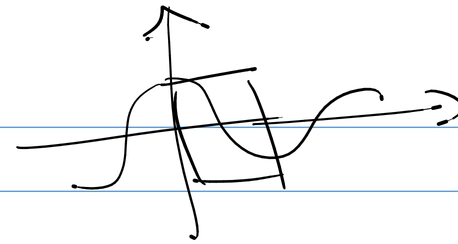
$y = \arcsin(x)$ is its inverse.

and

$$\boxed{\begin{gathered} \frac{d}{dx}\left[\arcsin(x)\right] = \frac{1}{\sqrt{1-x^2}} \\ \int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin(x) + C \end{gathered}}$$

plus

Do for each trig function..



$y = \cos(x) \quad 0 \le x \le \pi \quad -1 \le y \le 1$

$\rightarrow \quad y = \arccos(x) \quad -1 \le x \le 1 \quad 0 \le y \le \pi$

Table 4.4
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$D_x[\arccos(x)] = \dfrac{-1}{\sqrt{1-x^2}}$

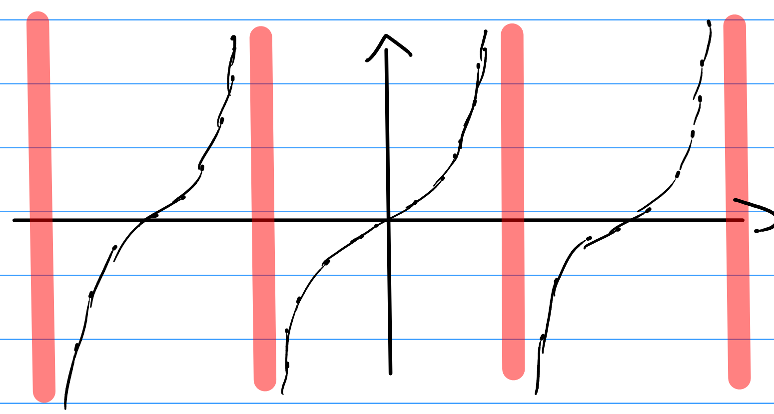
$D_x[\arctan(x)] = \dfrac{1}{1+x^2}$

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1}(x) + C$$

$y = \tan x$

$-\pi/2 < x < \pi/2$

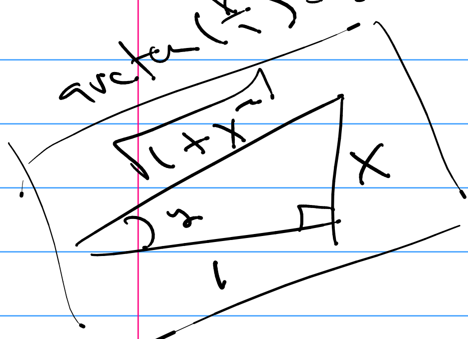
$-\infty < y < \infty$



$\rightarrow \quad y = \arctan(x) \quad -\infty < x < +\infty$

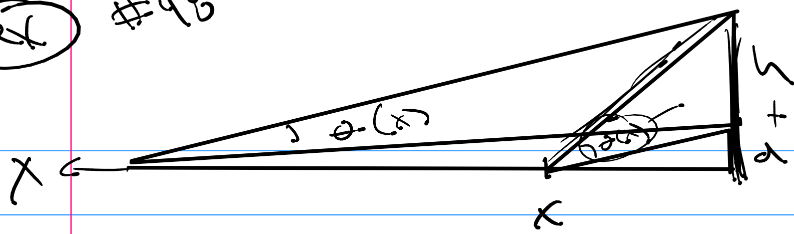
$-\pi/2 < y < \pi/2$

$\arctan\left(\frac{x}{1}\right) = y$



Means $\boxed{\tan(y) = x}$ Use to find $D_x[\arctan(x)]$

ex #48



Maximize θ?

derivative?

function?