Math 243

Q's

Note: ① Prelecture = extra credit

② automatic (3 day) extensions to webassign

③ Exam 1 → You work alone!

Integration Toolbox

→ ① Know the integrand?

(ex) $\int \sec x \tan x \, dx$?

(ex) $\int \frac{u^{1/2}}{1-u^{3/2}} du \sim \int \frac{1}{w} du$

② substitution → $w = 1 - u^{3/2}$

$dw = -3/2 \, u^{1/2} du$

③ by parts

7.3 Integrands with $\sqrt{c-x^2}$, $\sqrt{c+x^2}$, $\sqrt{x^2-c}$

(c is a constant)

(ex) $\int u\sqrt{3-u^2}\, du = -\frac{1}{2}\int w^{1/2} dw$

let $w = 3-u^2$

$dw = -2u\,du$

$= -\frac{1}{3} w^{3/2} + C$

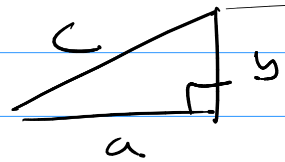
$= \boxed{-\frac{1}{3}(3-u^2)^{3/2} + C}$

ex $\int \sqrt{3-u^2}\,du$ Know: $\sqrt{(s)^2} = |s|$

Need trinomial for algebra

$(a+b)^2 = \boxed{a^2+2ab+b^2}$?

Know



$a^2+b^2=c^2$

trig: $\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$\boxed{\int \sqrt{3-u^2}\,du}$

Consider: $\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$\sqrt{3}\int \sqrt{1-\left(u/\sqrt{3}\right)^2}\,du$

let $\boxed{\frac{u}{\sqrt{3}} = \sin\theta}$ $-\pi/2 \le \theta \le \pi/2$

$\sqrt{1-\sin^2\theta} = \sqrt{\cos^2\theta} = |\cos\theta| = \cos\theta$

b/c $\frac{u}{\sqrt{3}} = \sin\theta \Rightarrow \boxed{\frac{1}{\sqrt{3}}\,du = \cos\theta\,d\theta}$

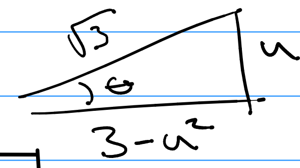
$\boxed{3\int \cos^2\theta\,d\theta}$ Know $\cos 2\theta = 2\cos^2\theta - 1$

$\frac{3}{2}\int (1+\cos 2\theta)\,d\theta = \frac{3}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right) + C$

$$= \frac{3}{2}\left(\theta + \frac{1}{2}\underbrace{\sin 2\theta}_{2\sin\theta\cos\theta}\right) + c$$

$$= \frac{3}{2}\theta + \frac{3}{2}\sin\theta\cos\theta + c \qquad \frac{u}{\sqrt{3}} = \sin\theta$$

$$= \frac{3}{2}\sin^{-1}\left(\frac{u}{\sqrt{3}}\right) + \frac{3}{2}\,\frac{u}{\sqrt{3}}\,\frac{3-u^2}{\sqrt{3}} + c$$



$$= \boxed{\frac{3}{2}\sin^{-1}\left(\frac{u}{\sqrt{3}}\right) + \frac{1}{2}u(3-u^2) + c}$$

Trigonometric Substitution

① $(a^2 - u^2)^{1/2}$ let $\boxed{u = a\sin\theta} \rightarrow du = ?$ $\quad -\pi/2 \le \theta \le \pi/2$

use: $\cos^2\theta = 1 - \sin^2\theta$

$(5 - u^2)^{1/2}$ let $\boxed{u = \sqrt{5}\sin\theta} \rightarrow du = ?$

② $(a^2 + u^2)^{1/2}$ let $\boxed{u = a\tan\theta} \rightarrow du = ?$

$-\pi/2 < \theta < \pi/2$

use: $1 + \tan^2\theta = \sec^2\theta$

③ $(u^2 - a^2)^{1/2}$ let $\boxed{u = a\sec\theta} \rightarrow du = ?$

$0 \le \theta < \pi/2$ or $\pi \le \theta < 3\pi/2$

use: $\sec^2\theta - 1 = \tan^2\theta$

ex. $\displaystyle\int \frac{x}{\sqrt{x^2-4}}\,dx$ matches #3 $\boxed{x = 2\sec\theta}$

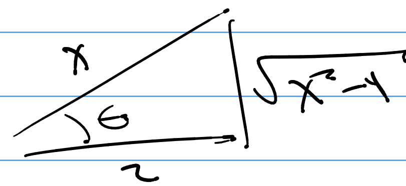
$dx = 2\sec\theta\tan\theta\,d\theta$

$$= \int \frac{2\sec\theta}{\sqrt{4\sec^2\theta - 4}}\, 2\sec\theta\tan\theta\,d\theta$$

$$= \int \frac{2\sec\theta}{2\cancel{\tan\theta}}\, 2\sec\theta\,\cancel{\tan\theta}\,d\theta$$

$$= 2\int \sec^2\theta\,d\theta = 2\tan\theta + C$$

Sub. $x = 2\sec\theta \Rightarrow \sec\theta = \frac{x}{2}$



$$= \boxed{\sqrt{x^2-4} + C}$$

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or $\displaystyle\int \frac{x}{\sqrt{x^2-4}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{u}}\,du = \frac{1}{2}\int u^{-1/2}\,du$

let $u = x^2-4$

$du = 2x\,dx$

$$= u^{1/2} + C$$

$$= \boxed{\sqrt{x^2-4} + C}$$

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ex $\displaystyle\int_0^{\pi/2} \frac{\cos t}{\sqrt{1+\sin^2 t}}\,dt$ vs $\displaystyle\int_0^{\pi/2} \frac{\cos t}{\sqrt{1-\sin^2 t}}\,dt$

let $u = \sin t$ $\quad t=0 \Rightarrow u=0$

$du = \cos t\,dt$ $\quad t=\pi/2 \Rightarrow u=1$

$$\boxed{\int_0^1 \frac{1}{\sqrt{1+u^2}}\,du}$$

$u = \tan\theta \quad du = \sec^2\theta\,d\theta$

$$\int_{u=0}^{u=1} \frac{\sec^2\theta\,d\theta}{\sqrt{1+\tan^2\theta}} = \int_{u=0}^{u=1} \sec\theta\,d\theta = \text{finish..}$$

$$= \int_0^{\pi/2} \frac{\cos t}{\cos t}\,dt$$

$$= \int_0^{\pi/2} 1\,dt = t\Big|_0^{\pi/2}$$

$$= \boxed{\pi/2}$$