Math 243

$\cancel{Q5}$ $\boxed{\int e^{2\theta}\sin(3\theta)\,d\theta}$

f	g'
f'	g

by parts $\int \underset{f}{(\text{function 1})}\underset{g'}{(\text{function 2})}\,dx$

vs sub $\int (\boxed{\text{function 1}})(\text{function 2})\,dx$

has an inside — deriv. & inside

$$\boxed{\int e^{m\theta}\sin(n\theta)\,d\theta} = -\frac{1}{n}e^{m\theta}\cos(n\theta) + \frac{m}{n}\boxed{\int e^{m\theta}\cos(n\theta)\,d\theta}$$

$f = e^{m\theta}$ $\quad g' = \sin(n\theta)$

$f' = me^{m\theta}\,d\theta$ $\quad g = -\frac{1}{n}\cos(n\theta)$

$\circledast$ $\int e^{m\theta}\cos(n\theta)\,d\theta = \boxed{\frac{1}{n}e^{m\theta}\sin(n\theta) - \frac{m}{n}\int e^{m\theta}\sin(n\theta)\,d\theta}$

$\rightarrow$ $f = e^{m\theta}$ $\quad g' = \cos(n\theta)$

$f' = me^{m\theta}\,d\theta$ $\quad g = \frac{1}{n}\sin(n\theta)$

$$\boxed{\int e^{m\theta}\sin(n\theta)\,d\theta} = -\frac{1}{n}e^{m\theta}\cos(n\theta) + \frac{m}{n}\left[\frac{1}{n}e^{m\theta}\sin(n\theta) - \frac{m}{n}\int e^{m\theta}\sin(n\theta)\,d\theta\right]$$

$$= -\frac{1}{n}e^{m\theta}\cos(n\theta) + \frac{m}{n^2}e^{m\theta}\sin(n\theta) - \frac{m^2}{n^2}\int e^{m\theta}\sin(n\theta)\,d\theta$$

take to left of =

$$\left(1+\frac{m^2}{n^2}\right)\boxed{\int e^{m\theta}\sin(n\theta)\,d\theta} = -\frac{1}{n}e^{m\theta}\cos(n\theta) + \frac{m}{n^2}e^{m\theta}\sin(n\theta) + C$$

$$\int e^{mt}\sin(nt)\,dt = \frac{-\frac{1}{n}e^{mt}\cos(nt) + \frac{m}{n^2}e^{mt}\sin(nt)}{\left(1 + m^2/n^2\right)} + C$$

$$\underline{\underline{\int \ln(x)\,dx}} = \int 1 \cdot \ln(x)\,dx = x\ln x - \int 1\,dx$$

$f = \ln x \quad g' = 1$

$f' = \frac{1}{x} \quad g = x$

$$= \boxed{x\ln x - x + C}$$

7.3 $\underline{\underline{\text{added}}}$

$(a^2 + x^2)^{1/2}$ let $x = a\sin\theta$

$(a^2 - x^2)^{1/2}$ let $x = a\tan\theta$

$(x^2 - a^2)^{1/2}$ let $x = a\sec\theta$

$\boxed{7.4}$ $\int \left(\text{rational function}\right) dx$

rational function $= \dfrac{\text{poly}}{\text{poly}}$

$\boxed{\text{Use some facts}}$

① deg (top) > deg (bottom)

$$\frac{P(x)}{Q(x)} = D(x) + \frac{M(x)}{Q(x)} \leftarrow \deg(M) < \deg(Q)$$

$$\frac{13}{5} = \overset{\text{div}}{\boxed{2}} + \frac{\overset{\text{mod}}{\boxed{3}}}{5} \qquad 12 = \boxed{2}\cdot 5 + 3$$

$$\boxed{\frac{x^3+x-1}{x+2}}$$

$$\|$$

$$x^2-2x+5-\frac{11}{x+2}$$

$$
\begin{array}{r|l}
 & x^2 - 2x + 5 \\
x+2 & x^3 + 0x^2 + x - 1 \\
 & x^3 + 2x^2 \\
\hline
 & -2x^2 + x \\
 & -2x^2 - 4x \\
\hline
 & 5x - 1 \\
 & 5x + 10 \\
\hline
 & -11
\end{array}
$$

So $\displaystyle\int \frac{x^3+x-1}{x+2}\,dx = \int\left(x^2-2x+5-\frac{11}{x+2}\right)dx$

$$= \boxed{\frac{1}{3}x^3 - x^2 + 5x - 11\ln|x+2| + C}$$

② **partial fractions**

$$\frac{1}{a}+\frac{1}{b}=\frac{b+a}{a\cdot b}$$

partial fraction decomposition

$$\frac{1}{2\cdot 3} = \frac{A}{2}+\frac{B}{3}$$

how to break across factors of denominator?

(ex) $\dfrac{M(x)}{Q(x)}$

① factors of $Q(x)$?

linear factors for real roots — (ex) $(ax+b)$

quadratic for complex — (ex) (ax^2+bx+c) $\quad x^2+4$

② Multiplicity of factors..

(ex) $(ax+b)^3$ ← root $-\frac{b}{a}$ is there 3 times.

ex $x^4 - 4x^3 + 8x^2 - 16x + 16$

$= (x^2 - 4x + 4)(x^2 + 4)$

$= \underline{(x-2)^2}\,\underline{(x^2+4)}$

linear factor of mult. 2 quad. factor of mult. 1

Partial Fraction Decomposition

$$\frac{M(x)}{Q(x)} = \frac{?}{\text{factor of } Q} + \frac{?}{\text{factor of } Q} + \frac{?}{\text{factor of } Q} + \cdots$$

(A) linear factor of mult. n ex $(ax+b)^n$

$$\to \frac{A_1}{ax+b} + \frac{A_2}{(ax+b)^2} + \cdots + \frac{A_n}{(ax+b)^n}$$

(B) quad. factor of mult. n ex $(ax^2+bx+c)^n$

$$\to \frac{A_1x+B_1}{(ax^2+bx+c)} + \frac{A_2x+B_2}{(ax^2+bx+c)^2} + \cdots + \frac{A_nx+B_n}{(ax^2+bx+c)^n}$$

ex
$$\frac{1}{(x-2)^2(x^2+4)} = \boxed{\frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{Cx+D}{(x^2+4)}}$$

A, B, C, D?

$$1 = A(x-2)(x^2+4) + B(x^2+4) + (Cx+D)(x-2)^2$$

$$0x^3 + 0x^2 + 0x + 1 = (\quad)x^3 + (\quad)x^2 + (\quad)x + (\quad)$$

Cont., ?!