

Math 243

Partial Fra. Decomposition

$$\frac{P(x)}{Q(x)} = \text{poly.} + \text{single rationals} + \dots$$

$$\int \frac{1}{x^4 - 4x^3 + 8x^2 - 16x + 16} dx =$$

Find $\text{root} \rightarrow (x - \text{root})$ is a factor.

① factor denominator.

possible rational roots $\frac{\text{factors } 16}{\text{factors } 1}$

check: 1, ② 4, 8, 16

$$\begin{array}{r}
 x^3 - 2x^2 + 4x - 8 \\
 x-2 \overline{) x^4 - 4x^3 + 8x^2 - 16x + 16} \\
 \underline{x^4 - 2x^3} \\
 -2x^3 + 8x^2 \\
 \underline{-2x^3 + 4x^2} \\
 4x^2 - 16x \\
 \underline{4x^2 - 8x} \\
 -8x + 16 \\
 \underline{-8x + 16} \\
 0
 \end{array}
 \quad Q(x) = (x-2)(x^3 - 2x^2 + 4x - 8)$$

etc $\int \frac{1}{(x-2)^2(x^2+4)} dx$

decomposition

$$\frac{1}{(x-2)^2(x^2+4)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{Cx+D}{x^2+4}$$

equality of num

$$1 = A(x-2)(x^2+4) + B(x^2+4) + (Cx+D)(x-2)^2$$

$$1 = A(x^3 - 2x^2 + 4x - 8) + B(x^2 + 4) + \underline{\underline{(Cx + D)(x^2 - 4x + 4)}}$$

$$1 = A(x^3 - 2x^2 + 4x - 8) + B(x^2 + 4) + C(x^3 - 4x^2 + 4x) + D(x^2 - 4x + 4)$$

x^3	$A + C = 0$	matrix →	$\left[\begin{array}{cccc c} 1 & 0 & 1 & 0 & 0 \\ -2 & 1 & -4 & 1 & 0 \\ 4 & 0 & 4 & -4 & 0 \\ -8 & 4 & 0 & 4 & 1 \end{array} \right]$
x^2	$-2A + B - 4C + D = 0$		
x	$4A + 4C - 4D = 0$		
const	$-8A + 4B + 4D = 1$		

$$\int \frac{-1/16}{(x-2)} + \frac{1/8}{(x-2)^2} + \frac{1/16x}{(x^2+4)} dx$$

= Finish

Integral Toolbox

7.5 (Strategy)

$$\int f(x) dx = \text{?}$$

① Recognize the Integrand. (Memory of "looking up" Integrand)

② Use Algebra / Trig to rewrite integrand.

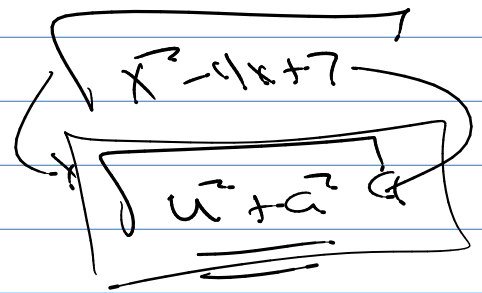
$$\int \frac{du}{u^2 + a^2} = \frac{1}{a} \tan^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{dx}{x^2 - 4x + 7} = \int \frac{1}{(x-2)^2 + 3} dx \quad \begin{array}{l} \text{let } u = x-2 \\ du = dx \end{array}$$

$$= \int \frac{1}{u^2 + (\sqrt{3})^2} du$$

② Substitution

③ Classify the problem type.
 trig? hyperbolic? radical?



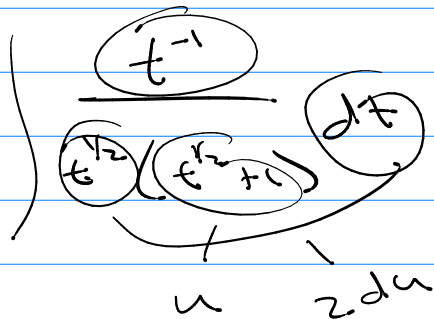
Rational? Products?
 \uparrow by parts?

④ Be creative

ex $\int \frac{dt}{t^2 + t\sqrt{t}} = \int \frac{dt}{t^2 + t^{3/2}}$

$$\frac{1}{t^2 + t^{3/2}} = \frac{1}{t(t + t^{1/2})}$$

$$= \frac{1}{t^{3/2}(t^{1/2} + 1)}$$



$$u = t^{1/2} + 1 \rightarrow (u-1)^2 = t$$

$$du = \frac{1}{2} t^{-1/2} dt$$

$$2 \int \frac{du}{(u-1)^2 u}$$