Math 243

Partial Frac. Decomposition

$$\frac{P(x)}{Q(x)} = \text{poly.} + \text{simple rationals} + \dots$$

$$\int \frac{1}{\boxed{x^4 - 4x^3 + 8x^2 - 16x + 16}}\,dx =$$

→ find root → $(x - \text{root})$ is a factor.

possible rational roots $\dfrac{\text{factors } 16}{\text{factors } 1}$

① factor denominator.

check: 1, 2, 4, 8, 16

$$\begin{array}{r}
x^3 - 2x^2 + 4x - 8 \\
x - 2 \overline{)\, x^4 - 4x^3 + 8x^2 - 16x + 16} \\
\underline{x^4 - 2x^3} \qquad\qquad\qquad\quad \\
-2x^3 + 8x^2 \qquad\qquad\quad \\
\underline{-2x^3 + 4x^2} \qquad\qquad\quad \\
4x^2 - 16x \qquad\quad \\
\underline{4x^2 - 8x} \qquad\quad \\
-8x + 16 \\
\underline{-8x + 16} \\
0
\end{array}$$

$$Q(x) = (x-2)(x^3 - 2x^2 + 4x - 8)$$

etc

$$\int \frac{1}{(x-2)^2(x^2+4)}\,dx$$

Decomposition

$$\frac{1}{(x-2)^2(x^2+4)} = \frac{A}{(x-2)} + \frac{B}{(x-2)^2} + \frac{Cx+D}{(x^2+4)}$$

equality of num

$$1 = A(x-2)(x^2+4) + B(x^2+4) + (Cx+D)(x-2)^2$$

$$1 = A(x^3-2x^2+4x-8) + B(x^2+4) + (Cx+D)(x^2-4x+4)$$

$$1 = A(x^3-2x^2+4x-8) + B(x^2+4) + C(x^3-4x^2+4x) + D(x^2-4x+4)$$

$$\begin{array}{l|l} x^3 & A + C = 0 \\ x^2 & -2A + B - 4C + D = 0 \\ x & 4A + 4C - 4D = 0 \\ \text{const} & -8A + 4B + 4D = 1 \end{array}$$

Matrix

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ -2 & 1 & -4 & 1 & 0 \\ 4 & 0 & 4 & -4 & 0 \\ -8 & 4 & 0 & 4 & 1 \end{array}\right]$$

$$\int \frac{-1/16}{(x-2)} + \frac{1/8}{(x-2)^2} + \frac{1/16\, x}{(x^2+4)}\, dx$$

$$= \boxed{\text{Finish}}$$

Integration Toolbox   7.5 (Strategy)

$$\int f(x)\,dx = ?$$

(0) Recognize the Integrand. (Memory & "looking up" Integrands)

(1) Use $\boxed{\text{Algebra / Trig}}$ to rewrite integrand.

ex $$\int \frac{du}{u^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{dx}{x^2-4x+7} = \int \frac{1}{(x-2)^2+3}\,dx$$

let $u = x-2$
$du = dx$

$$= \int \frac{1}{u^2 + (\sqrt{3})^2}\, du$$

② Substitution

③ Classify the problem type.

trig? hyperbolic? radical? $\sqrt{x^2 - 4x + 7} \to \boxed{\sqrt{u^2 + a^2}}$

rational? products? ← by parts?

④ Be creative

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ex) $$\int \frac{dt}{t^2 + t\sqrt{t}} = \int \frac{dt}{t^2 + t^{3/2}}$$

$$\frac{1}{t^2 + t^{3/2}} = \frac{1}{t(t + t^{1/2})}$$

$$= \frac{1}{t^{3/2}(t^{1/2} + 1)}$$

$$\int \frac{t^{-1}}{t^{1/2}(t^{1/2}+1)}\, dt$$

u $\quad$ $2du$

$u = t^{1/2} + 1 \to (u-1)^2 = t$

$du = \frac{1}{2} t^{-1/2}\, dt$

$$2 \int \frac{du}{(u-1)^2 u}$$