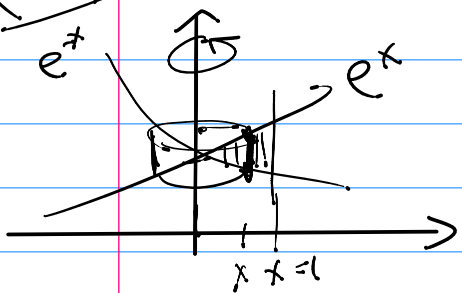
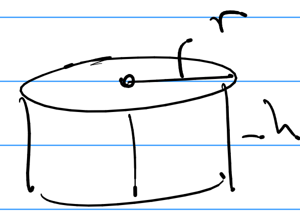
Math 243

Q's 7.1 #62 $y = e^x$ $y = e^{-x}$ $x = 1$ about y-axis



$$2\pi \int_{x=0}^{x=1} x(e^x - e^{-x})\,dx$$



$$2\pi\left[\underbrace{\int_0^1 xe^x\,dx}_{\text{use by parts}} \underbrace{- \int_0^1 xe^{-x}\,dx}_{\text{use by parts}}\right]$$

(*)

$(e^x - e^{-x})$

$2\pi x$

(*) $\displaystyle\int_0^1 xe^{-x}\,dx = -xe^{-x}\Big|_0^1 \mp \int_0^1 e^{-x}\,dx$

$f = x \qquad g' = e^{-x}$

$f' = 1 \qquad g = -e^{-x}$

$= \boxed{(-xe^{-x})\Big|_0^1} + \boxed{(-e^{-x})\Big|_0^1}$

$= \boxed{(-e^{-1}) - (0)}$

$+ \boxed{(-e^{-1}) - (-1)}$

$= \boxed{1 - \frac{2}{e}}$

ex $\int \frac{dt}{t^2+t^{3/2}}$

$\boxed{\text{remove radicals by substitution}}$ $\boxed{\text{idea:}}$ $\boxed{x^{1/3}}$, $x^{-3/4}$

$\sqrt[3]{\ }$, $\sqrt[4]{\ }$ lcm $= 12$

let $u = x^{1/12} \Rightarrow u^{12} = x$

$12u^{11}\,du = dx$

So $x^{1/3} = (u^{12})^{1/3} = u^4$ $x^{-3/4} = (u^{12})^{-3/4} = u^{-9}$

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ex $\int \frac{dt}{t^2+t^{3/2}} = \int \frac{2u\,du}{u^4+u^3} = 2\int \frac{du}{u^3+u^2}$

$\boxed{\text{remove radicals}}$ let $u = t^{1/2} \to u^2 = t$ $t^{3/2} = (u^2)^{3/2} = u^3$

$2u\,du = dt$

$= 2\int \frac{du}{u^2(u+1)}$?? $\frac{1}{(u)^2(u+1)} = \frac{\overset{-1}{A}}{u} + \frac{\overset{1}{B}}{u^2} + \frac{\overset{1}{C}}{u+1}$??

Now: $1 = A(u(u+1)) + B(u+1) + C(u^2)$

$0u^2+0u+1 = A(u^2+u) + B(u+1) + C(u^2)$

u^2 $A + C = 0$

u $A + B = 0$

const $B = 1$

by substitution $\boxed{B=1,\ A=-1,\ C=1}$

as Matrix: $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right]$ $R_2 - R_1$ $=$ new R_2 $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right]$

$r_3 - r_2 =$ New r_3

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$A + C = 0 \to A + 1 = 0 \quad A = -1$

$B - C = 0 \quad B - 1 = 0$

$B = 1$

$C = 1$

Now:

$$2\int \frac{du}{u^2(u+1)} = 2\int \left(\frac{-1}{u} + \frac{1}{u^2} + \frac{1}{u+1} \right) du$$

$$= 2\left[-\ln|u| - \frac{1}{u} + \ln|u+1| \right] + C$$

$\boxed{\text{Sub}}$ $u = \sqrt{t}$ $\quad = 2\left[-\ln\sqrt{t} - \frac{1}{\sqrt{t}} + \ln(\sqrt{t}+1) \right] + C$

Strategy (F) $\boxed{\text{Tables of Integrals}}$

$$\int \frac{\sqrt{3x^2+1}}{x^2}\,dx = \sqrt{3}\int \frac{\sqrt{x^2 + 1/3}}{x^2}\,dx \quad (\sqrt{1/3})^2 \quad a = \sqrt{1/3}$$

ref. Forms $\sqrt{a^2+u^2}$ $\quad a > 0$

\#24 $\displaystyle\int \frac{\sqrt{a^2+u^2}}{u^2}\,du = -\frac{\sqrt{a^2+u^2}}{u} + \ln(u + \sqrt{a^2+u^2}) + C$

$\boxed{\text{C.A.S.}}$