

# Math 243

Q5 (ex)  $\int \frac{dx}{\sqrt{x^2+2x+5}}$

Sketch:  $\sqrt{x^2+2x+(1)^2} + 5 - (1)^2$   
 $= (x+1)^2 + 4$

$\sqrt{a^2-x^2}$	$x = a \sin \theta$
$\sqrt{a^2+x^2}$	$x = a \tan \theta$
$\sqrt{x^2-a^2}$	$x = a \sec \theta$

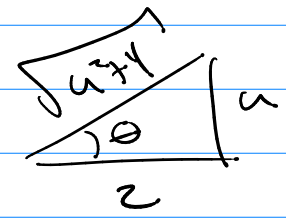
$\int \frac{dx}{\sqrt{(x+1)^2 + 4}} = \int \frac{du}{\sqrt{u^2 + (2)^2}}$      let  $u = 2 \tan \theta$   
 $du = 2 \sec^2 \theta d\theta$

let  $u = x+1$   
 $du = dx$       $= \int \frac{2 \sec^2 \theta d\theta}{\sqrt{(2 \tan \theta)^2 + 4}}$

$= \int \frac{2 \sec^2 \theta d\theta}{\sqrt{4 \tan^2 \theta + 4}} = \int \frac{\sec^2 \theta d\theta}{\sqrt{\tan^2 \theta + 1}} = \int \sec \theta d\theta$   
 $\downarrow$   
 $\sec^2 \theta$

$= \ln |\sec \theta + \tan \theta| + C$

back to  $u$ 's      $u = 2 \tan \theta$



$= \ln \left| \frac{\sqrt{u^2+4}}{2} + \frac{u}{2} \right| + C$

back to  $x$ 's      $u = x+1$

$= \ln \left| \frac{\sqrt{(x+1)^2+4}}{2} + \frac{x+1}{2} \right| + C$

$\ln(a \cdot b) = \ln(a) + \ln(b)$

$$\int \frac{dx}{\sqrt{x^2+2x+5}} = \ln \left| \sqrt{(x+1)^2+4} + x+1 \right| + C$$


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tables  
 $\int$  Integrals  $\rightarrow$  Use? Expand integrals you "know"

⑤ C.A.S. (Computer Algebra Systems)

$\rightarrow$  Computational Mathematics

① Numeric (ex 7.07)

② Octave (floats, ints, booleans)

③ Symbolic

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How to use software?

$\rightarrow$  Maxima > Mathematica

When to use software?

① you can do the problem  
 $\rightarrow$  you just don't want to

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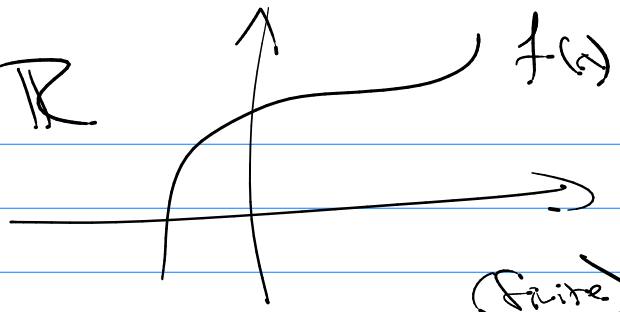
② Experiment

make simple problems to do extra exercises.

$$\int \frac{dx}{\sqrt{x^2+x+1}}$$

Functions:

$$f: \mathbb{R} \rightarrow \mathbb{R}$$



Algebraic Functions:

$f(x) =$  expression uses Algebra (Finite)

Elementary Functions:

$f(x) =$  expression uses (Finite) Algebraic (+)  $e^x$ ,  $\ln(x)$ , Trig, Hyperbolic

$\text{erf}(x)$   $\xrightarrow{\text{can't do this}}$   $\mathbb{R}$

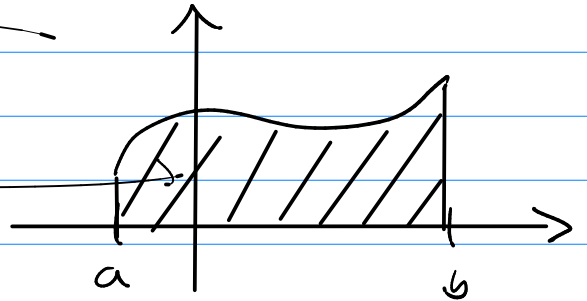
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## Approximate Integration

uses

①  $\int_a^b f(x) dx$

area



$\rightarrow \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*) \Delta x$

$$\Delta x = \frac{b-a}{n}$$

$x_i^*$  is an in the  $i^{\text{th}}$  interval

② Fund. th<sup>m</sup>  $\int_a^b f(x) dx = \underline{\underline{F(b) - F(a)}}$

where  $\frac{d}{dx} [F(x)] = f(x)$