Math 293

$\boxed{Q's}$ $\int \frac{x^2+x+1}{(x^2+1)^2}\,dx$

$$\frac{x^2+x+1}{(x^2+1)^2} = \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+1)^2}$$

Numerator $x^2+x+1 = (Ax+B)(x^2+1) + Cx+D$

$$x^2+x+1 = A(x^3+x) + B(x^2+1) + C(x) + D(1)$$

x^3: $A=0$ so $\frac{x^2+x+1}{(x^2+1)^2} = \frac{1}{x^2+1} + \frac{x}{(x^2+1)^2}$

x^2: $B=1$

x: $A+C=1 \to C=1$

const: $B+D=1 \to D=0$

$\therefore \int \frac{x^2+x+1}{(x^2+1)^2}\,dx = \int \frac{1}{x^2+1}\,dx + \int \frac{x}{(x^2+1)^2}\,dx$ let $u=x^2+1$, $du=2x\,dx$

$= \arctan(x) + \frac{1}{2}\int u^{-2}\,du$

$= \arctan(x) - \frac{1}{2}u^{-1} + C = \boxed{\arctan(x) - \frac{1}{2(x^2+1)} + C}$

check: $\frac{d}{dx}\left[\arctan(x) - \frac{1}{2}(x^2+1)^{-1} + C\right]$

$= \frac{1}{x^2+1} + \frac{1}{2}(x^2+1)^{-2}(2x)$

$= \frac{1}{x^2+1} + \frac{x}{(x^2+1)^2} = \frac{(x^2+1)+x}{(x^2+1)^2} = \frac{x^2+x+1}{(x^2+1)^2}$

7.4 #31

$$\int \frac{35}{x^3-27}\,dx = 35\int \frac{1}{x^3-27}\,dx$$

Factoring: (you should be able to ...)

$$ax^2+bx+c$$

$$(a^2+2ab+b^2) = (a+b)^2$$

$$a^2-b^2 = (a+b)(a-b)$$

$$a^3+b^3 = (a+b)(a^2-ab+b^2)$$

$$a^3-b^3 = (a-b)(a^2+ab+b^2)$$

by grouping quad term

$$\rightarrow 35\int \frac{1}{x^3-3^3}\,dx = 35\int \frac{1}{(x-3)(x^2+3x+9)}\,dx$$

So

$$\frac{1}{(x-3)(x^2+3x+9)} = \boxed{\frac{A}{x-3}+\frac{Bx+C}{x^2+3x+9}}$$

Numerators

$$1 = A(x^2+3x+9)+(Bx+C)(x-3)$$

$$1 = A(x^2+3x+9)+B(x^2-3x)+C(x-3)$$

x^2: $A+B=0$ $(\rightarrow A=-B)$

x: $3A-3B+C=0$

const: $9A-3C=1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 3 & -3 & 1 & 0 \\ 9 & 0 & -3 & 1 \end{array}\right]$$

Solve

try: $\boxed{\text{sub } 3}$ $(-6B + C = 0)3$

$$-9B - 3C = 1$$

$+$

$$-27B = 1 \qquad B = -1/27 \qquad \text{etc}$$

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up to $\boxed{7.7}$ $\boxed{\text{Find}}$

① $\int f(x)\,dx = \boxed{\text{expression}}$

② $\int_a^b f(x)\,dx = F(b) - F(a)$

where $F(x) = \boxed{\text{expression}}$

$\boxed{7.7}$ Tools to use when

$$\int_a^b f(x)\,dx = \boxed{\text{can't find an expression}}$$

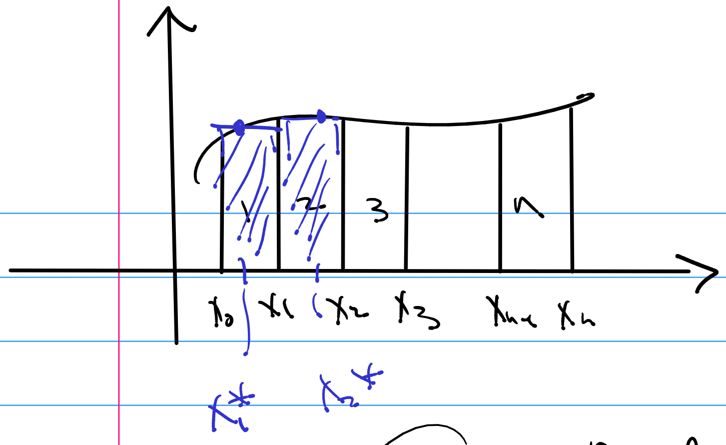
$\boxed{7.7}$ $\boxed{\text{Approximating Integration}}$

$$\int_a^b f(x)\,dx = \lim_{n \to +\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x \qquad \Delta x = \frac{b-a}{n}$$

n-partitions

a, b; $n=1$, $n=2$, $n=3$, ..., n

$x_0 \quad x_1 \quad x_2 \quad x_3 \quad \ldots \quad x_{n-1} \quad x_n$



n - rectangles

area(rectangle) = length · width

→ assume $\lim_{n\to+\infty} \sum_{i=1}^{n} f(x_i^*)\cdot\Delta x$ can't be done.

$\boxed{\text{approximate}}$ let n = fixed value

$$\int_a^b f(x)\,dx \approx \sum_{i=1}^{n} f(x_i^*)\Delta x$$

bunch of arithmetic to do

$\Delta = \frac{b-a}{n}$ $\quad f(x_i^*) = ?$ list of numbers

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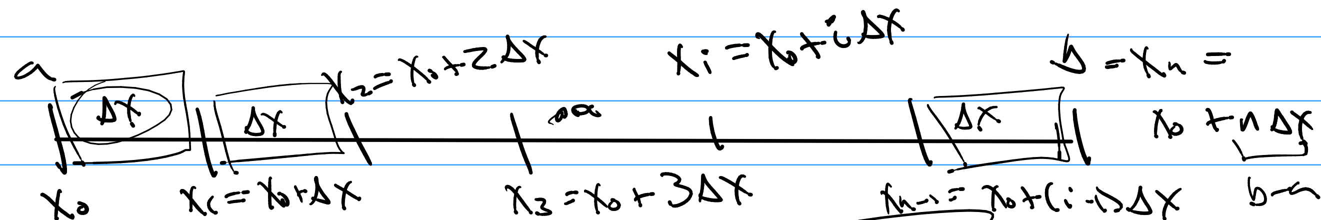
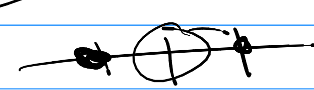
$\boxed{\text{Help for the arithmetic as well as picking } x_i^*}$

$\boxed{\text{techniques}}$ Approx Integrals are based (named) on



always choose n = number $\quad \Delta x = \boxed{\frac{b-a}{n}} =$



tech #1

use $\sum_{i=1}^{n} f(x_i^*)\Delta x = \Delta x \boxed{\sum_{i=1}^{n} f(x_i^*)}$

left end point approximation:

① x's $= \{x_0, x_1, x_2, \dots, x_{n-1}\} = \{a+(i-1)\Delta x\}$

② $f(x_i^*) = \{f(x_0), f(x_1), f(x_2), \dots, f(x_{n-1})\}$

$\boxed{f(a+(i-1)\Delta x)}$ all heights.

③ add all heights

④ Area $\approx \Delta x \sum_{i=1}^{n} f(x_i^*)$

tech #2

right end point approx

① pick x's $= \{a+\Delta x, a+2\Delta x, \dots, a+n\Delta x = b\}$

② $f(x_i^*) = \{f(a+\Delta x), f(a+2\Delta x), \dots\}$ = heights

$f(x_i^*) = a + i\Delta x \quad i=1,2,\dots,n$

③ add the heights

④ Area $\approx \Delta x \sum_{i=1}^{n} f(x_i^*)$

Note: $\Delta x \sum_{i=1}^{n} f(x_i^*) = \frac{(b-a)}{n}\left[f(x_1^*) + f(x_2^*) + \dots + f(x_n^*)\right]$

Notice: $\text{Area} \approx (b-a)\left[\frac{f(x_1^*) + f(x_2^*) + \dots + f(x_n^*)}{n}\right]$