Math 243

Q's | 7.3.026 | $\int \frac{x^2}{(15+4x-4x^2)^{3/2}}\,dx$

trig sub

$\sqrt{a^2-x^2}$	$x = a\sin\theta$
$\sqrt{a^2+x^2}$	$x = a\tan\theta$
$\sqrt{x^2-a^2}$	$x = a\sec\theta$

$(a+b)^2$

Scratch: $15 + 4x - 4x^2$

$-4x^2 + 4x \quad + 15$

$-4\left[x^2 - x + (-\tfrac{1}{2})^2 - \tfrac{1}{4}\right] + 15$

$-4\left\{(x-\tfrac{1}{2})^2 - \tfrac{1}{4}\right\} + 15$

So $15+4x-4x^2 = 16 - 4(x-\tfrac{1}{2})^2$

$$\therefore \int \frac{x^2}{(15+4x-4x^2)^{3/2}}\,dx = \int \frac{x^2}{(16-4(x-\frac{1}{2})^2)^{3/2}}\,dx$$

$$= \frac{1}{8}\int \frac{x^2}{(4-(x-\frac{1}{2})^2)^{3/2}}\,dx = \int \frac{\boxed{(u+\frac{1}{2})^2}}{(2^2-u^2)^{3/2}}\,du$$

let $u = x - \tfrac{1}{2}$
$du = dx$

let $u = 2\sin\theta$

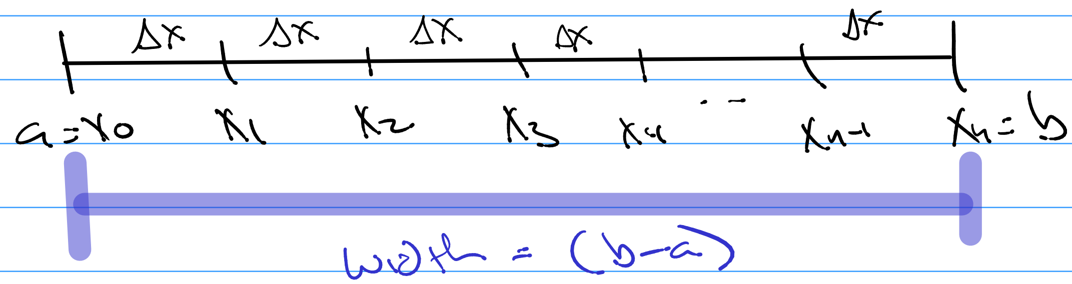
= Continue

$$? = \boxed{\int \frac{u^2}{(2^2-u^2)^{3/2}}\,du} + \boxed{\int \frac{u}{(2^2-u^2)^{3/2}}\,du} + \frac{1}{4}\int \frac{1}{(2^2-u^2)^{3/2}}\,du$$

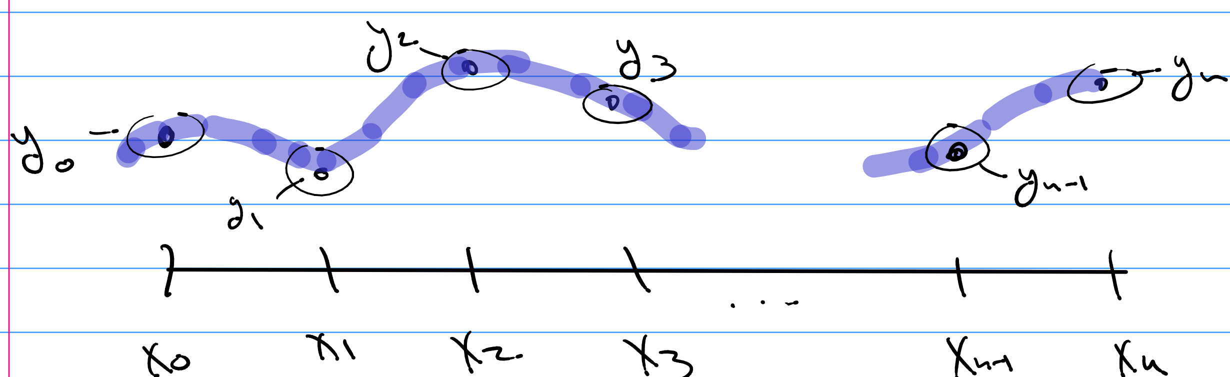
7.7 $\int_a^b f(x)\,dx \approx \sum_{i=1}^{n} f(x_i^*)\,\Delta x \qquad \Delta x = \frac{b-a}{n}$

$\qquad\qquad\qquad\qquad\qquad\qquad \parallel$

Id $\int_a^b f(x)\,dx \approx (b-a)\frac{\sum_{i=1}^{n} f(x_i)}{n}$

① 

② $y_0 = f(x_0),\ y_1 = f(x_1),\ y_3 = f(x_3),\ \dots,\ y_n = f(x_n)$



Have: $x_0, x_1, x_2, \dots, x_n$

$y_0, y_1, y_2, \dots, y_n$

③ **Approx Integration Tech.**

A) Left endpoint approx

$$\text{Area} \approx (b-a)\cdot \text{average}(y_0, y_1, \dots, y_{n-1})$$

B) Right End point Approx.

$$\text{Area} \approx (b-a)\ \text{average}(y_1, y_2, \dots, y_n)$$

C) <u>Midpoint</u>   Say intervals $= n/2$

<u>Here</u>

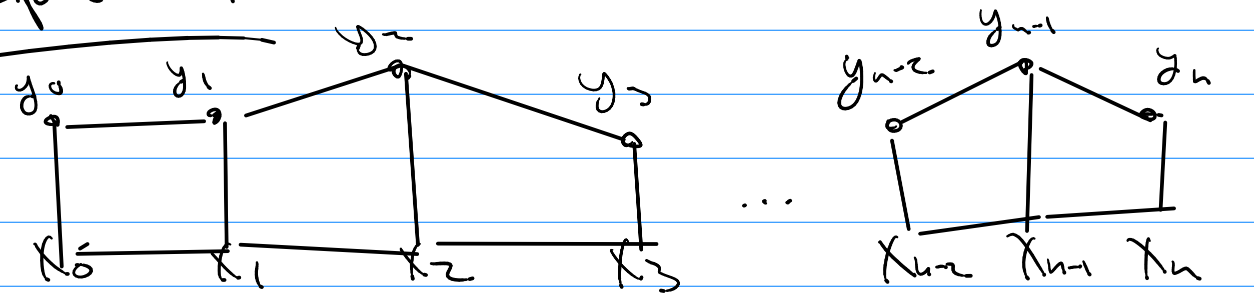
$$\begin{array}{ccccccccccc} x_0 & x_1 & x_2 & x_3 & x_4 & \cdots & x_{n-2} & x_{n-1} & x_n \\ y_0 & y_1 & y_2 & y_3 & y_4 & \cdots & y_{n-2} & y_{n-1} & y_n \end{array}$$

<u>Midpoints</u>

$$\text{Area} \approx (b-a)\ \frac{y_1 + y_3 + y_5 + \dots y_{n-1}}{n/2}$$

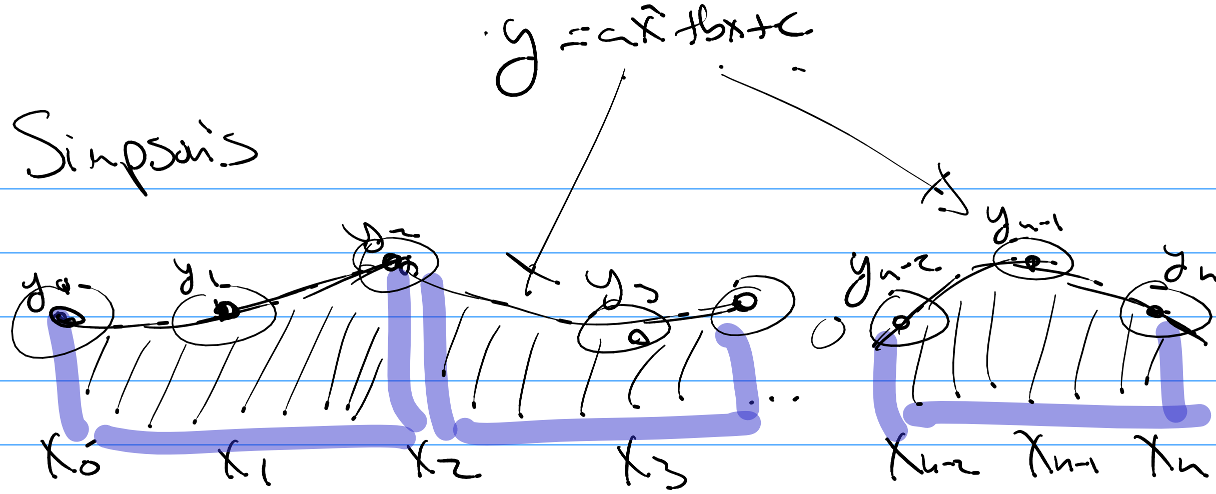
trap/Simp rules use weighted averages

D) <u>trapezoidal</u>



$$\text{Area} \approx (b-a)\left[\frac{y_0 + 2y_1 + 2y_2 + 2y_3 + \dots + 2y_{n-1} + y_n}{2n}\right]$$

E) Simpson's



$$\text{Area} \approx (b-a)\left[\frac{y_0 + 4y_1 + 2y_2 + 4y_3 + \dots + 2y_{n-2} + 4y_{n-1} + y_n}{3n}\right]$$

$$\int_a^b f(x)\,dx = \text{Approx} + \boxed{\text{error}}$$

Error Estimates:

n = # of intervals (always make even)

K_i = max of i^{th} deriv.

$$|f^{(i)}(x)| \le K_i \text{ on } [a,b]$$

$$|E_{Left}| \le \frac{K_1 (b-a)^2}{2n}$$

$$|E_R| \le \frac{K_1 (b-a)^2}{2n}$$

$$|E_T| \le \frac{K_2 (b-a)^3}{12n^2}$$

$$|E_S| \leq \frac{K_4 (b-a)^5}{180 n^4}$$