Math 243

Q's $\int_0^1 \boxed{\cos(x^2)}\,dx$ $y = f(x)$ $n = 4$

trapezoidal

$n=1$, $n=2$, $n=3$, $n=4$

$0 \quad 1/4 \quad 1/2 \quad 3/4 \quad 1$ x's

y's $\cos(0)$, $\cos(1/16)$, $\cos(1/4)$, $\cos(9/16)$, $\cos(1)$

area $\approx (1)\dfrac{\cos(0) + 2\cos(1/16) + 2\cos(1/4) + 2\cos(9/16) + \cos(1)}{8}$

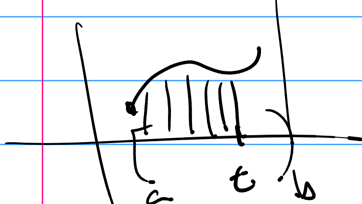
Midpt x's $1/8 \quad 3/8 \quad 5/8 \quad 7/8$

y's $\cos\left(\frac{1}{64}\right)$, $\cos(9/64)$, $\cos(25/64)$, $\cos(49/64)$

area $\approx (1)\dfrac{\cos(1/64) + \cdots + \cos(49/64)}{4}$

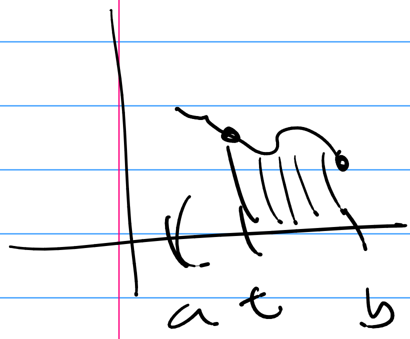
7.8 $\int_a^b f(x)\,dx$ where f is disc. @ a, b, or some c between.

Type I $\int_a^b f(x)\,dx$ but domain is $[a, b)$



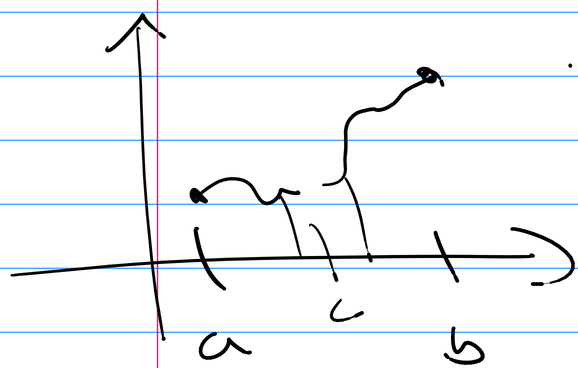
$$\int_a^b f(x)\,dx = \lim_{t \to b^-} \int_a^t f(x)\,dx$$

type 2 $\int_a^b f(x)\,dx$ but domain is $(a, b]$



$$\int_a^b f(x)\,dx = \lim_{t \to a^+} \int_t^b f(x)\,dx$$

type 3 $\int_a^b f(x)\,dx$ but domain is $[a, c) \cup (c, b]$



$$\int_a^b f(x)\,dx = \underbrace{\int_a^c f(x)\,dx}_{\text{type 1}} + \underbrace{\int_c^b f(x)\,dx}_{\text{type 2}}$$

---

ex $$\int_0^2 \frac{1}{(x-1)^2}\,dx = \int_0^1 \frac{1}{(x-1)^2}\,dx + \int_1^2 \frac{1}{(x-1)^2}\,dx$$

not defined @ $x=1$

all wrong!

vs You miss this fact!

$$\int_0^2 \frac{1}{(x-1)^2}\,dx = \int_{-1}^1 \frac{1}{u^2}\,du = \left[-\frac{1}{u}\right]\Big|_{-1}^{1}$$

let $u = x-1$

$du = dx$

$$= (-1 - 1) = -2$$

Correct $$\int_0^2 \frac{1}{(x-1)^2}\,dx = \int_0^1 \frac{1}{(x-1)^2}\,dx + \int_1^2 \frac{1}{(x-1)^2}\,dx$$

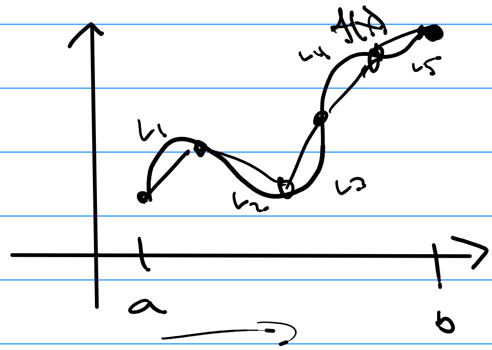
$$\int_0^1 \frac{1}{(x-1)^2}\,dx = \lim_{t\to 1^-}\int_0^t \frac{1}{(x-1)^2}\,dx = \lim_{t\to 1^-} \left.\frac{-1}{x-1}\right|_0^t$$

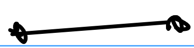
$$= \lim_{t\to 1^-}\left[\overset{\to +\infty}{\cancel{\frac{-1}{t-1}}} - 1\right] = +\infty \quad \boxed{\text{divergent}}$$

$\boxed{\text{Ch 8}}$ Applications of Integration

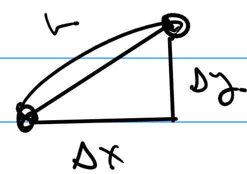
$$\int f(x)\,dx \quad \text{or} \quad \int_a^b f(x)\,dx$$

$\boxed{8.1}$ $\boxed{\text{Arc lengths}}$





length of straight line



$$L \approx \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

$$L_i \approx \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2}\,\Delta x$$

$$\text{arc length} \approx \sum_{i=1}^{n} L_i \qquad \text{as } n \to +\infty \qquad \Delta x \to dx$$

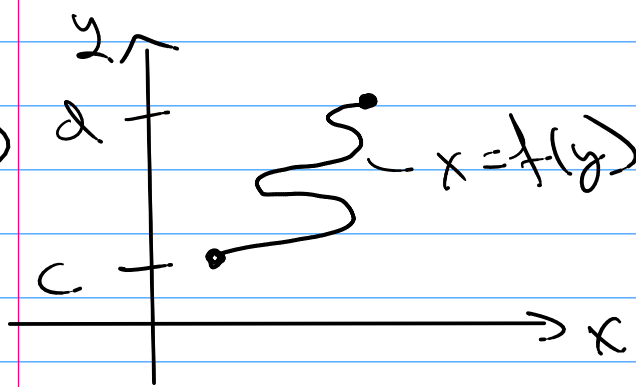
$$\frac{\Delta y}{\Delta x} \to f'(x)$$

$$\text{arc length} = \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{1 + (y')^2}\,dx$$

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx = \int_a^b \sqrt{1 + (f'(x))^2}\,dx$$

Note: $f(x)\,dx$ variable x is by my choice.

vs



$$L = \int_c^d \sqrt{1 + (f'(y))^2}\, dy$$

ex

$$\boxed{y = \ln(\cos x)} \qquad 0 \le x \le \pi/3 \qquad \frac{dy}{dx} = \frac{1}{\cos x}\cdot(-\sin x)$$

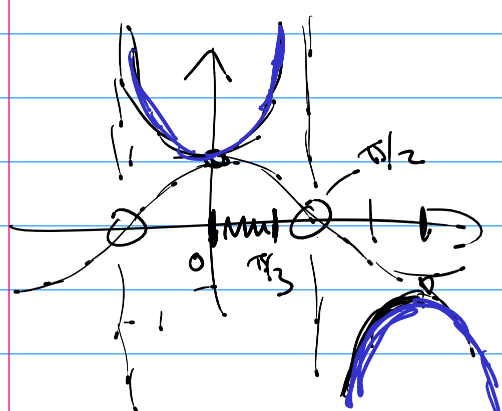
$$= -\tan x$$

$$L = \int_0^{\pi/3} \sqrt{1 + (f'(x))^2}\, dx$$

$$L = \int_0^{\pi/3} \sqrt{1 + \tan^2 x}\, dx$$

Note $\sqrt{\square^2} = |\square|$

$$L = \int_0^{\pi/3} \sqrt{\sec^2 x}\, dx = \int_0^{\pi/3} |\sec x|\, dx$$



$$= \int_0^{\pi/3} \sec x\, dx$$

$$= \ln|\sec x + \tan x| \Big|_0^{\pi/3}$$

$$= \boxed{\text{etc}}$$

ex $f(x) = \frac{1}{4}x^2 - \frac{1}{2}\ln x \qquad 1 \le x \le 2$

$$L = \int_1^2 \sqrt{1 + \left(\frac{1}{2}x - \frac{1}{2x}\right)^2}\, dx$$

$$L = \int_1^2 \sqrt{\frac{x^2}{4} + \frac{1}{2} + \frac{1}{4x^2}}\, dx$$