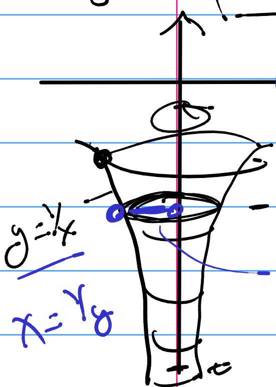
Math 243

8.2 $\boxed{\#23}$, #27



$r = \boxed{\left(0 - \frac{1}{y}\right)}$

$f = \frac{1}{y}$

$f' = -\frac{1}{y^2}$

$$V = \int_{-\infty}^{-1} \pi r^2\, dy = \int_{-\infty}^{-1} \frac{\pi}{y^2}\, dy = \lim_{t \to -\infty} \left[\pi \int_t^{-1} y^{-2}\, dy \right]$$

$$= \lim_{t \to -\infty} \left[\pi \left(-\frac{1}{y} \right) \Big|_t^{-1} \right]$$

$$= \pi \lim_{t \to -\infty} \left[1 + \cancel{\frac{1}{t}}^{\,0} \right] = \boxed{\pi} \text{ units}^3$$

$$SA = \int_{-\infty}^{-1} 2\pi r \sqrt{1 + (f')^2}\, dy = \int_{-\infty}^{-1} 2\pi \left(-\frac{1}{y} \right) \sqrt{1 + \frac{1}{y^4}}\, dy$$

$$SA = -2\pi \int_{-\infty}^{-1} \left(\frac{1}{y} \right) \sqrt{1 + \frac{1}{y^4}}\, dy$$

$$SA = -2\pi \int_{-\infty}^{-1} \frac{\sqrt{y^4 + 1}}{y^3}\, dy = \pi \int_1^{\infty} \frac{\sqrt{u^2 + 1}}{u^2}\, du$$

let $\boxed{u = y^2}$

$du = 2y\, dy$

table: $\sqrt{u^2 + a^2}$

#24 table of integrals (ref pg. 6)

$$SA = \pi \int_1^{\infty} \frac{\sqrt{u^2+1}}{u^2}\, du = \pi \left[-\frac{\sqrt{u^2+1}}{u} + \ln\left(u + \sqrt{u^2+1}\right) \right]_1^{\infty}$$

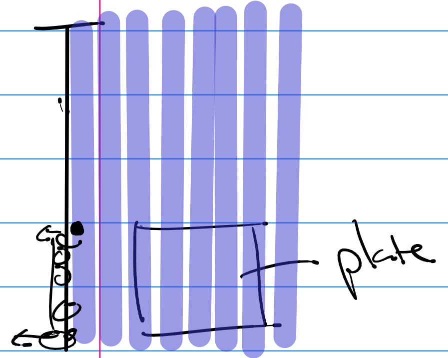
$$SA = \pi \lim_{t \to +\infty} \left[-\frac{\sqrt{u^2+1}}{u} + \ln\left(u + \sqrt{u^2+1}\right) \right] \Big|_1^t$$

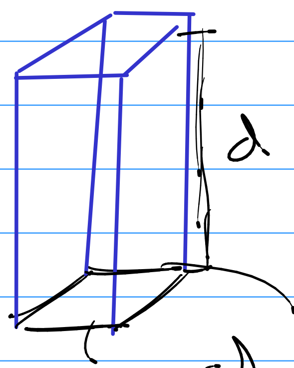
$$= +\infty$$

8.3 Physics (Applied problems with integrals)

$$\int_a^b f(x)\,dx$$

Fluid Pressure





$F = \text{mass} \cdot \text{accel}$

mass of fluid — gravity

area $= A$ Volume $= Ad$

ρ = mass density

$$\rho = \frac{\text{mass}}{\text{Vol}}$$

So $\boxed{F = \rho \cdot Ad \cdot g}$

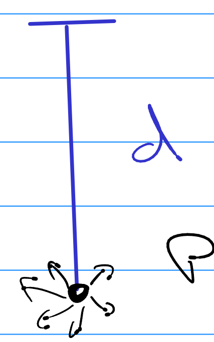
$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} \quad \text{(So)} \Rightarrow \text{Fluid Pressure} = \rho\, d\, g$$

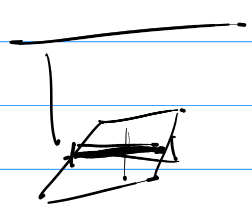
Note: $\rho \cdot g = \dfrac{\text{mass}}{\text{vol}} \cdot g = \dfrac{\text{mass} \cdot g}{\text{vol}} = \dfrac{\text{weight}}{\text{Vol}} =$ weight density

$\delta = \rho g$ is weight density

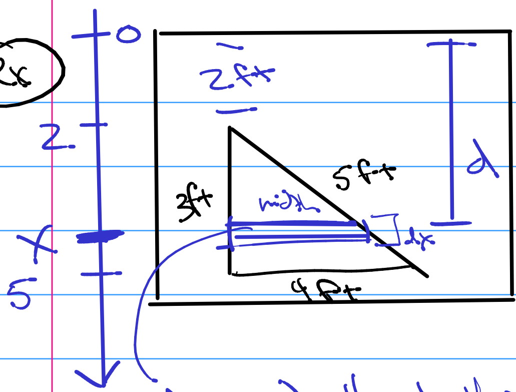
$$F = \rho \cdot Ad \cdot g = \delta A d$$

$$P = \rho d g = \delta d$$





Ex



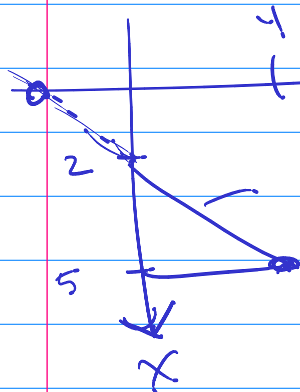
Force = Sum of F_i as we slice from top to bottom of plate.

$$F_i = \delta A_i d_i$$

d_i — depth at this slice

A_i — area at this slice

$A_i = (\text{width})\, dx$, $\quad d_i = x$



$$w = f(x) = \frac{4}{3}(x-2)$$

(2, 0) $\qquad$ (5, 4)

$$w = \frac{4}{3}(x-2)$$