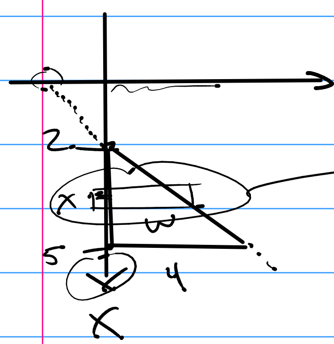
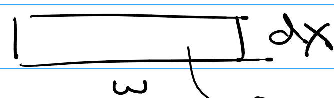
Math 243

8.3 Fluid Pressure / Force

$F_i = (\delta)(\text{Area})(\text{Depth})$ — δ: weight density





force on this slice

$(\delta)(w\,dx)(x)$

last class $w = \frac{4}{3}(x-2)$

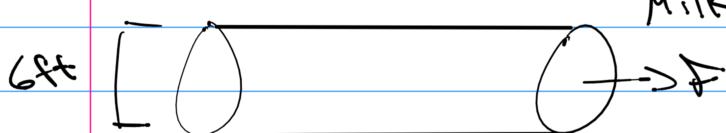
Force = sum of all F_i

$$F = \int_2^5 (\delta)\left(\tfrac{4}{3}(x-2)\,dx\right)(x) = \tfrac{4}{3}\delta \int_2^5 (x^2-2x)\,dx$$

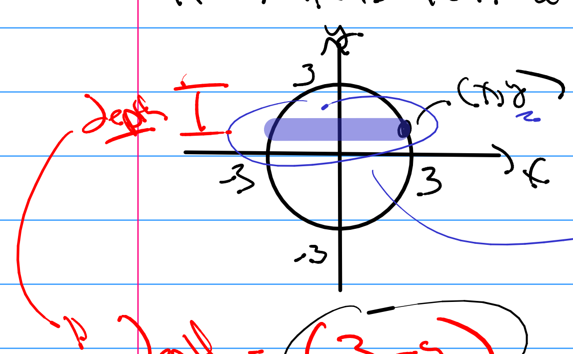
$$F = \tfrac{4}{3}\delta \cdot 18 = \boxed{24\,\delta}$$

#12 in textbook

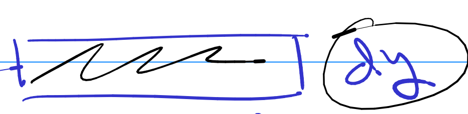
Milk: $\delta = 64.6 \ \text{lb}/\text{ft}^3$



if tank is full what is force on end?



$x^2+y^2 = 3^2 \ \leadsto \ x = \sqrt{9-y^2}$



$w = 2x$

$w = 2\sqrt{9-y^2}$

depth $= (3-y)$

$$F_i = (\delta)(\text{Area})(\text{depth}) = (\delta)(2\sqrt{9-y^2}\,dy)(3-y)$$

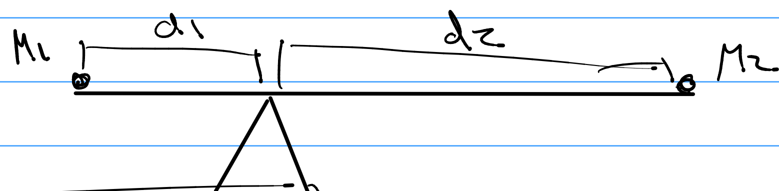
$$F = 2\delta \int_{-3}^{3} \left[(3-y)\sqrt{9-y^2}\right] dy$$

$$= 2\delta \left[3\int_{-3}^{3} \sqrt{9-y^2}\, dy - \int_{-3}^{3} y\sqrt{9-y^2}\, dy \right]$$

$\frac{9}{2}\pi$

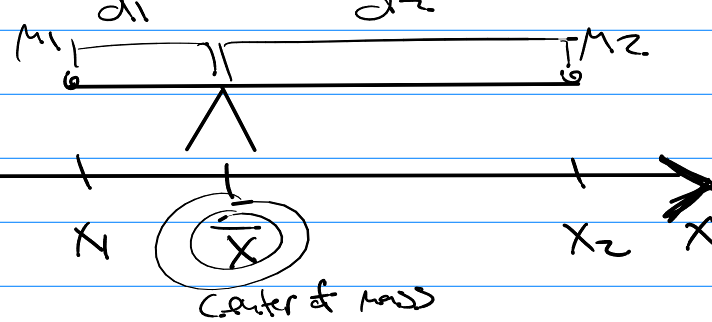
$$F = 27\pi\delta \approx \boxed{5480 \text{ ft lb}}$$

Levers

① 

$$m_1 d_1 = m_2 d_2$$

② Apply a coord. system



$$m_1 d_1 = m_2 d_2$$

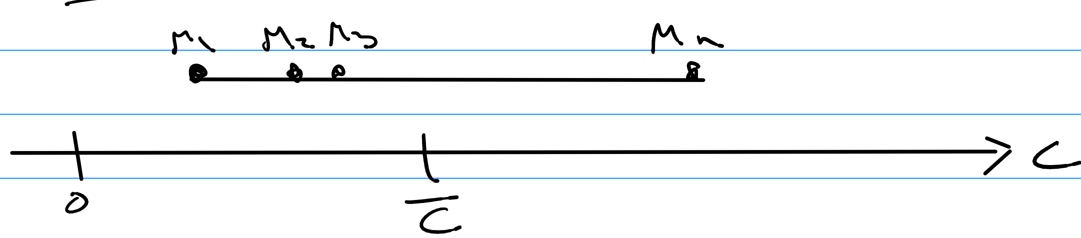
$$m_1(\bar{x} - x_1) = m_2(x_2 - \bar{x})$$

$$m_1\bar{x} - x_1 m_1 = m_2 x_2 - m_2\bar{x}$$

$$\bar{x} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

moment of mass 2

③ Several Masses $m_1, m_2, m_3, \ldots, m_n$



$$\bar{c} = \frac{m_1c_1 + m_2c_2 + m_3c_3 + \cdots + m_nc_n}{m_1 + m_2 + \cdots + m_n}$$

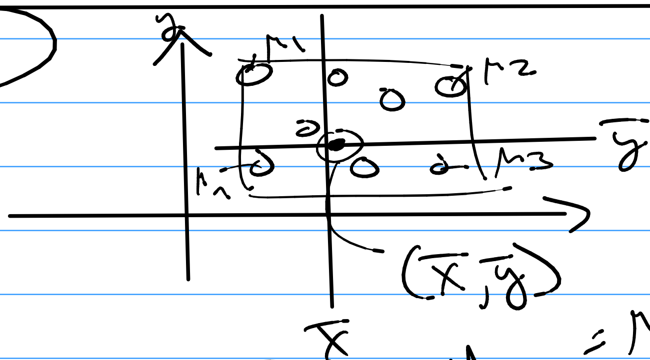
$$\boxed{\bar{c} = \frac{\sum_{i=1}^{n} m_ic_i}{\sum_{i=1}^{n} m_i}}$$

(4) Continuous bar

Mass function $m(x)$ = mass @ point x

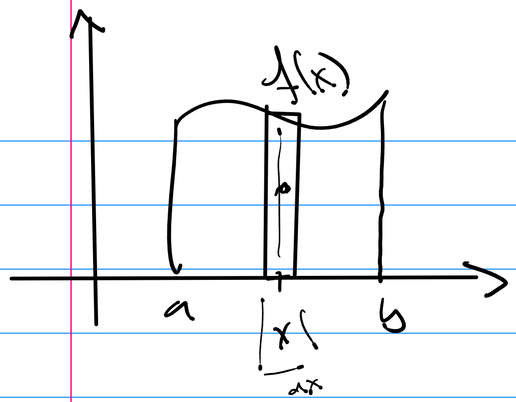
$$\sum m_ix_i \to \int_a^b m(x)dx \cdot x$$

2D



$$\bar{x} = \frac{\sum_{i=1}^{n} m_ix_i}{\sum_{i=1}^{n} m_i} \qquad \sum_{i=1}^{n} m_ix_i = M_{y\text{-axis}} = M_y$$

$$\bar{y} = \frac{\sum_{i=1}^{n} m_iy_i}{\sum_{i=1}^{n} m_i} \qquad \sum_{i=1}^{n} m_iy_i = M_{x\text{-axis}} = M_x$$



across the entire sheet it has a constant density $\rho = \frac{\text{mass}}{\text{area}}$

$$\text{total mass} = \rho \cdot \text{Area} = \rho \int_a^b f(x)\,dx$$

$$\bar{x} \left(= \frac{\sum_{i=1}^{n} (M_i)(x_i)}{\sum_{i=1}^{n} M_i} \right) \qquad \bar{x} = \frac{\int_a^b \overbrace{(\rho f(x)dx)}^{\text{mass}}\overbrace{(x)}^{\text{coord}}}{\rho \int_a^b f(x)dx}$$

$$\boxed{\bar{x} = \frac{\int_a^b x f(x)dx}{\int_a^b f(x)dx}}$$

$$\bar{y} \left(= \frac{\sum_{i=1}^{n} (M_i)(y_i)}{\sum_{i=1}^{n} M_i} \right) \qquad \bar{y} = \frac{\int_a^b (\rho f(x)dx)(\frac{1}{2} f(x))}{\rho \int_a^b f(x)dx}$$

$$\boxed{\bar{y} = \frac{\frac{1}{2}\int_a^b (f(x))^2 dx}{\int_a^b f(x)dx}}$$