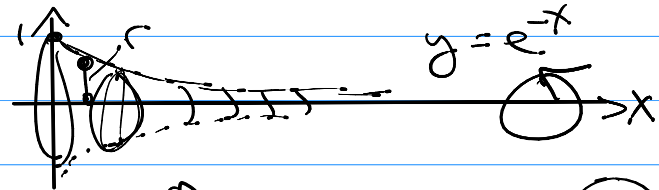
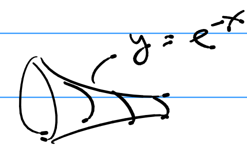
Math 243

Q5 | 8.2 #28





$$V = \int_0^\infty \pi r^2\,dx = \pi \int_0^\infty (e^{-x})^2\,dx = \pi \int_0^{\infty} e^{-2x}\,dx$$

$$V = \lim_{t\to+\infty} \pi \int_0^t e^{-2x}\,dx = \lim_{t\to+\infty}\left[-\frac{\pi}{2} e^{-2x}\Big|_0^t\right]$$

$$V = \lim_{t\to+\infty}\left(-\frac{\pi}{2}\underbrace{e^{-2t}}_{\to 0} + \frac{\pi}{2}\right) = \boxed{\frac{\pi}{2}}$$

$$SA = \int_0^\infty 2\pi r\sqrt{1+(f')^2}\,dx \qquad r = f(x) = e^{-x}, \quad f' = -e^{-x}$$

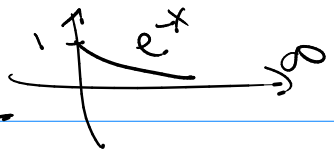
$$SA = \int_0^\infty 2\pi (e^{-x})\sqrt{1+(e^{-x})^2}\,dx$$

let $\boxed{u = e^{-x}}$ $\quad du = -e^{-x}\,dx$

$$= -\int_1^0 2\pi\sqrt{1+u^2}\,du \qquad \lim_{x\to+\infty} e^{-x} = 0$$

$$= 2\pi\int_0^1 \sqrt{1+u^2}\,du = \boxed{\text{finish}}$$

trig sub or table or **CAS**

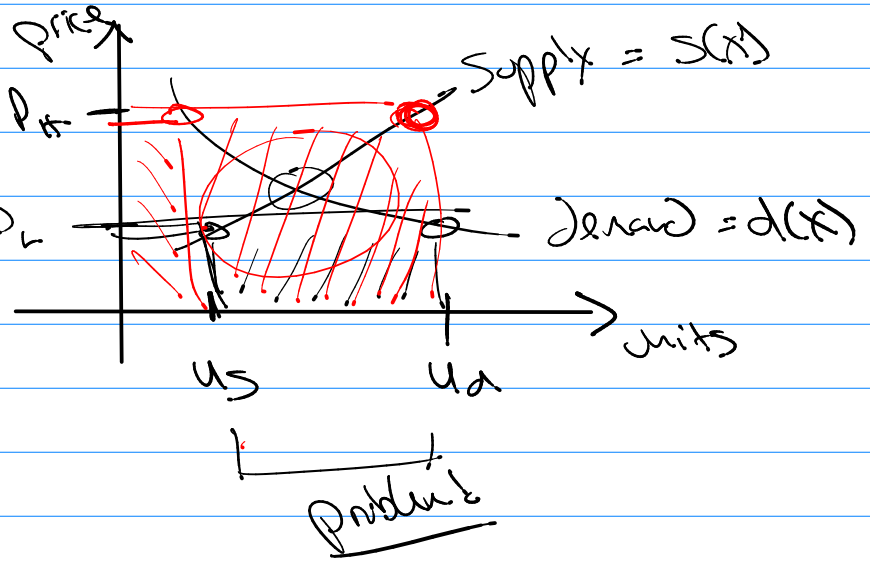
Area? 

$$A = \int_0^{\infty} e^{-x}\,dx = \lim_{t\to+\infty}\left[\int_0^t e^{-x}\,dx\right]$$

$$= \lim_{t\to+\infty}\left[-e^{-x}\Big|_0^t\right] = \lim_{t\to+\infty}\left[-e^{-t}+1\right]$$

$$= 1$$

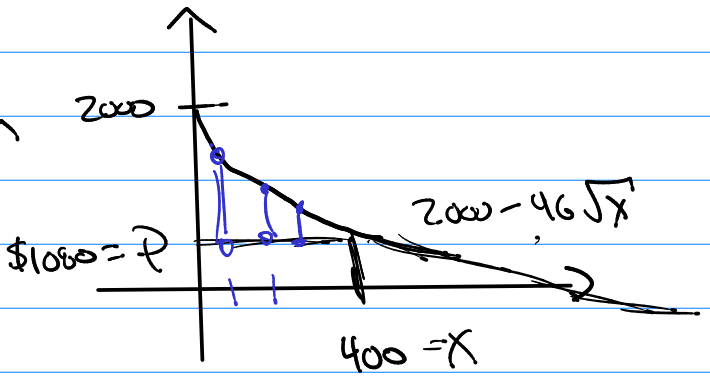
Econ



(ex) Demand function:

price $= d(x) = 2000 - 46\sqrt{x}$

↑ units



Revenue $= \$1080 \cdot 400 = \$432{,}000$

$p = 2000 - 46\sqrt{400}$

$= 2000 - 920 = 1080$

Consumer Surplus $= \int_0^{400} (2000 - 46\sqrt{x}) - 1080\,dx$

$$= \int_0^{400} (920 - 46x^{1/2})\,dx = 920x - \tfrac{2}{3}46x^{3/2}\Big|_0^{400}$$

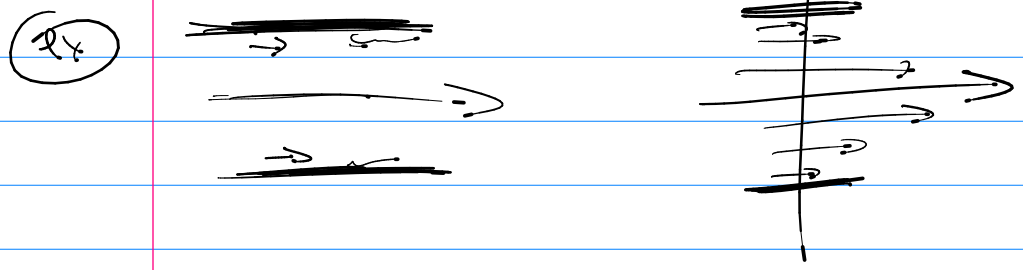
Consumer Surplus $= 920 \cdot 400 - \frac{92}{3}(400)^{3/2}$

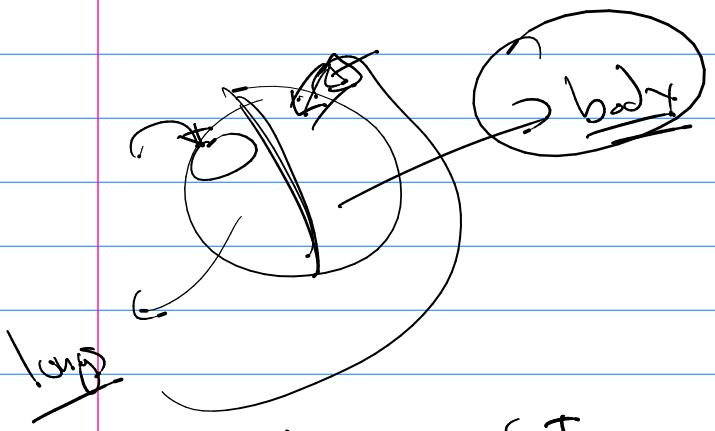
$= 92\left[4000 - \frac{8000}{3}\right]$

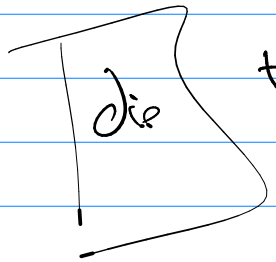
$= \frac{92}{3}\left[12000 - 8000\right] = \frac{920400}{3}$

$\approx 31 \cdot 4000 \approx \boxed{\$124{,}000}$

Biological



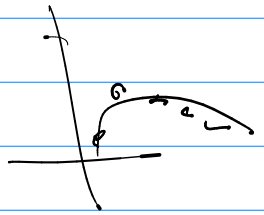




$C(t)$
0
0
.1
.3
.5
1
3
4
.5
.1
.1
0

$t = T$

$$A = F\int_0^T C(t)\,dt$$



$$F = \frac{A}{\int_0^T C(t)\,dt}$$

lt

$A = 5.5$ mg $n = 9$

18 sec

t = 0 s	0 mg/L
t = 2	4.1
t = 4	8.9
t = 6	8.5
t = 8	6.7
t = 10	4.3
t = 12	2.5
t = 14	1.2
t = 16	0.2
t = 18	0

$$\text{left} = 18\left(\frac{0 + 4.1 + 8.9 + 8.5 + \dots + 0.2}{9}\right)$$

$$\text{right} = 18\left(\frac{4.1 + 8.9 + \dots + 0.2 + 0}{9}\right)$$

$$\text{mid} = 18\left(\frac{4.1 + 8.5 + 4.3 + 1.2}{4}\right)$$

$$\text{trap} = 18\left(\frac{0 + 2\cdot 4.1 + 2\cdot 8.9 + \dots + 2\cdot 0.2 + 0}{18}\right)$$

$$\text{simp} = 18\left(\frac{0 + 4\cdot 4.1 + 2\cdot 8.9 + \dots + 4\cdot 1.2 + 0.2}{24}\right)$$